Modeling Individual Behaviors in Social and Crowd Dynamics *Looking for the Black Swan*

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Kinetic description of emerging challenges
in multiscale problems of natural sciences
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Plan of the Lecture

1. Towards a Mathematical Theory of Living Systems

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1 - Towards a Mathematical Theory of Living Systems

Five Common Features and Sources of Complexity

1. Ability to express a strategy: Living entities are capable to develop specific ***strategies*** and *organization abilities* that depend on the state of the surrounding environment.

2. Heterogeneity: The ability to express a strategy is not the same for all entities: *Heterogeneity* characterizes a great part of living systems.

3. Learning ability: Living systems receive inputs from their environments and have the ability to learn from past experience. Therefore their strategic ability and the characteristics of interactions evolve in time.

4. Nonlinear Interactions: Interactions nonlinearly additive and involve immediate neighbors, but in some cases also distant particles. Indeed, living systems have the ability to communicate and can possibly choose different observation paths.

5. Darwinian selection and time as a key variable: All living systems are evolutionary as birth processes can generate individuals more fitted to the environment, who can generate new individuals even more fitted.

1 - Towards a Mathematical Theory of Living Systems

Technical Features of Complex Systems

• ***Multiscale aspects:*** The study of complex living systems always needs a *multiscale approach*, where the dynamics at the large scale needs to be properly related to the dynamics at the low scales. For instance, the functions expressed by a cell are determined by the dynamics at the molecular (genetic) level. This feature characterizes also the dynamics of vehicles and animals, where the mechanical system is linked to individual behaviors.

• ***Time varying role of the environment:*** The environment surrounding a living system evolves in time, in several cases also due to the interaction with the inner system. Therefore the interaction rules and, in some cases, also the number of components of a living system evolve in time.

• ***Large deviations;*** Emerging behaviors are often related to large deviations although the qualitative behaviors are often reproduced, namely small deviations in the input create large deviations in the output.

1 - Towards a Mathematical Theory of Living Systems

Strategy

- The overall system is subdivided into ***functional subsystems*** constituted by entities, called ***active particles***, whose individual state is called ***activity***
- The state of each functional subsystem is defined by a ***probability distribution over the micro-scale state***, which includes position, velocity, and activity variables, which represent the strategies expressed heterogeneously by each individual;
- Interactions are modeled by ***evolutionary stochastic games***, where the state of the interacting particles and the output of the interactions are known in probability;
- The evolution of the probability distribution is obtained by a balance of particles within elementary volume of the space of the microscopic states, where the dynamics of inflow and outflow of particles is related to interactions at the microscopic scale.

1 - Towards a Mathematical Theory of Living Systems

Representation for space distributed systems in each node

Consider active particles in a node for functional subsystems labeled by the subscript i.

• The description of the overall state of the system is delivered by the ***generalized one-particle distribution function***

$$f_i = f_i(t, \mathbf{x}, \mathbf{v}, u) = f_i(t, \mathbf{w}) \; : \quad [0, T] \times \Omega \times D_{\mathbf{v}} \times D_u \to \mathbf{R}_+,$$

such that $f_i(t, \mathbf{x}, \mathbf{v}, u)\, d\mathbf{x}\, d\mathbf{v}\, du = f_i(t, \mathbf{w})\, d\mathbf{w}$ denotes the number of active particles whose state, at time t, is in the interval $[\mathbf{w}, \mathbf{w} + d\mathbf{w}]$ of the i-th subsystem.

• $\mathbf{w} = \{\mathbf{x}, \mathbf{v}, u\}$ is an element of the ***space of the microscopic states***.

• $\mathbf{x}$ and $\mathbf{v}$ Represent the ***mechanical variables***, whenever these have a physical meaning: in some cases they are vanishing variables, while in networks, the space is substituted by nodes.

• The ***activity variable*** can be a vector.

1 - Towards a Mathematical Theory of Living Systems

Stochastic Games Living entities, at each interaction, *play a game* with an output that technically depends on their strategy often related to surviving and adaptation abilities, namely to an individual or collective search for fitness. The output of the game generally is not deterministic even when a causality principle is identified.

- **Test** particles of the i-th functional subsystem with microscopic state, at time t, delivered by the variable $(\mathbf{x}, \mathbf{v}, u) := \mathbf{w}$, whose distribution function is $f_i = f_i(t, \mathbf{x}, \mathbf{v}, u) = f_i(t, \mathbf{w})$. The test particle is assumed to be representative of the whole system.

- **Field** particles of the k-th functional subsystem with microscopic state, at time t, defined by the variable $(\mathbf{x}^*, \mathbf{v}^*, u^*) := \mathbf{w}^*$, whose distribution function is $f_k = f_k(t, \mathbf{x}^*, \mathbf{v}^*, u^*) = f_k(t, \mathbf{w}^*)$.

- **Candidate** particles, of the h-th functional subsystem, with microscopic state, at time t, defined by the variable $(\mathbf{x}_*, \mathbf{v}_*, u_*) := \mathbf{w}_*$, whose distribution function is $f_h = f_h(t, \mathbf{x}_*, \mathbf{v}_*, u_*) = f_h(t, \mathbf{w}_*)$.

Stochastic Games

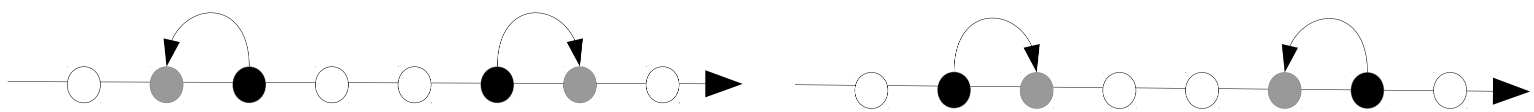
1. **Competitive (dissent):** When one of the interacting particle increases its status by taking advantage of the other, obliging the latter to decrease it. Therefore the competition brings advantage to only one of the two. This type of interaction has the effect of increasing the difference between the states of interacting particles, due to a kind of driving back effect.

2. **Cooperative (consensus):** When the interacting particles exchange their status, one by increasing it and the other one by decreasing it. Therefore, the interacting active particles show a trend to share their micro-state. Such type of interaction leads to a decrease of the difference between the interacting particles' states, due to a sort of dragging effect.

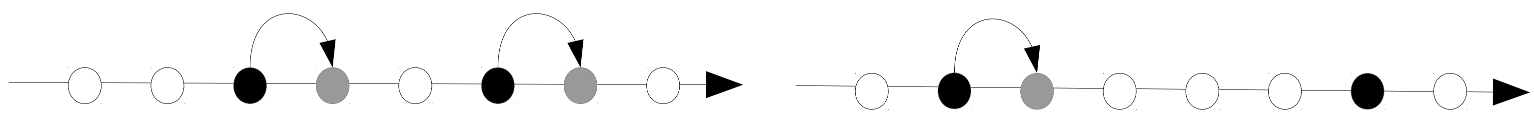
3. **Learning:** One of the two modifies, independently from the other, the micro-state, in the sense that it learns by reducing the distance between them.

4. **Hiding-chasing:** One of the two attempts to increase the overall distance from the other, which attempts to reduce it.

Stochastic Games Pictorial illustration of (a) competitive,(b) cooperative, (c) hiding-chasing and (d) learning game dynamics between two active particles. Black and grey bullets denote, respectively, the pre- and post-interaction states of the particles.

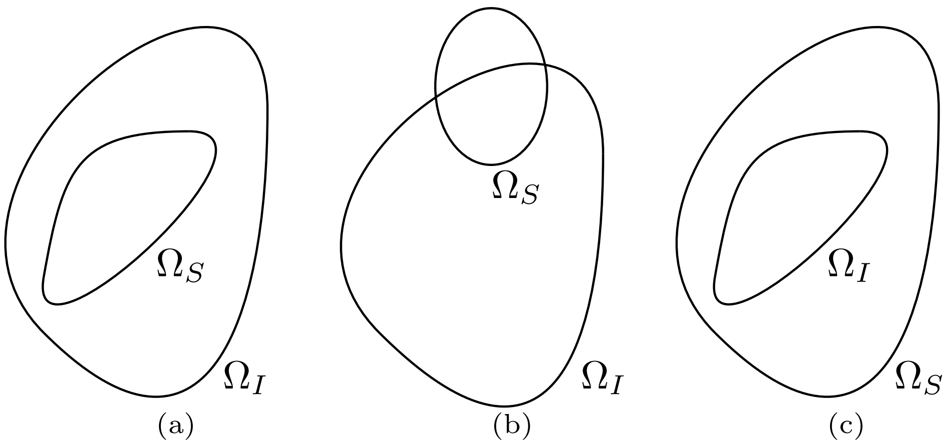


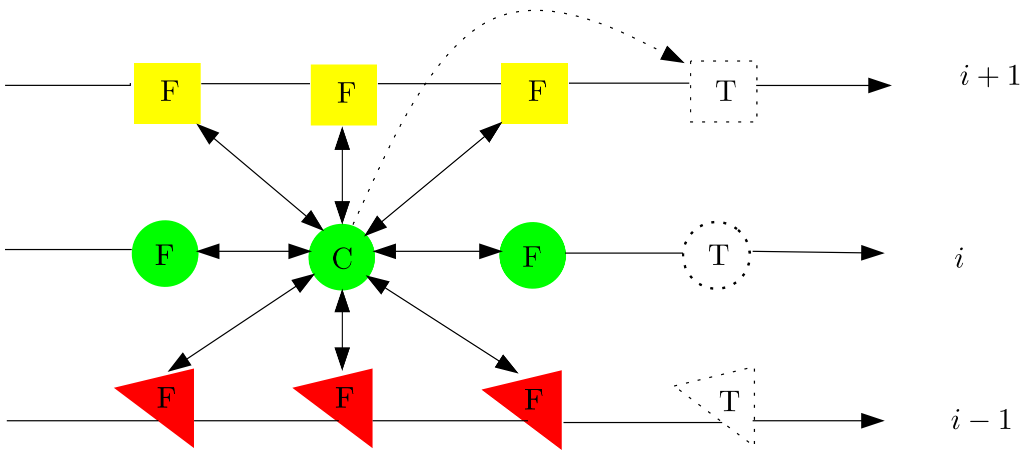
(a) Competition *(b) Cooperation*



(c) Hiding-chasing *(d) Learning*

Stochastic Games Representations





Mathematical Structures: Models with Space Dynamics

H.1. Candidate or test particles in $\mathbf{x}$, interact with the field particles in the interaction domain $\mathbf{x}^* \in \Omega$. Interactions are weighted by the ***interaction rates*** $\eta_{hk}[\mathbf{f}]$ and $\mu_{hk}[\mathbf{f}]$ supposed to depend on the local distribution function in the position of the field particles.

H.2. A candidate particle modifies its state according to the probability density: $\mathcal{C}_{hk}^i[\mathbf{f}](\mathbf{v}_* \to \mathbf{v},\, u_* \to u|\mathbf{w}_*, \mathbf{w})$, which denotes the probability density that a candidate particles of the h-subsystems with state $\mathbf{w}_* = \{\mathbf{x}_*, \mathbf{v}_*, u_*\}$ reaches the state $\{\mathbf{v}, u\}$ in the i-th subsystem after an interaction with the field particles of the k-subsystems with state $\mathbf{w}^* = \{\mathbf{x}^*, \mathbf{v}^*, u^*\}$.

H.3. A candidate particle, in $\mathbf{x}$, can proliferate, due to encounters with field particles in $\mathbf{x}^*$, with rate $\mu_{hk}\mathcal{P}_{hk}^i$, which denotes the proliferation rate into the functional subsystem i, due the encounter of particles belonging the functional subsystems h and k. Destructive events can occur only within the same functional subsystem with rate $\mu_{ik}\mathcal{D}_{ik}$.

1 - Towards a Mathematical Theory of Living Systems

Balance within the space of microscopic states and Structures

Variation rate of the number of active particles

$= \textit{Inlet flux rate caused by conservative interactions}$

$+\textit{Inlet flux rate caused by proliferative interactions}$

$-\textit{Outlet flux rate caused by destructive interactions}$

$-\textit{Outlet flux rate caused by conservative interactions},$

where the inlet flux includes the dynamics of mutations.

This flow-chart corresponds to the following structure:

$$(\partial_t + \mathbf{v} \cdot \partial_{\mathbf{x}})\; f_i(t, \mathbf{x}, \mathbf{v}, u) = \big(J_i^C - J_i^L + J_i^P - J_i^D\big)[\mathbf{f}](t, \mathbf{x}, \mathbf{v}, u),$$

where the various terms J_i can be formally expressed, consistently with the definition of η, μ, $\mathcal{C}$, $\mathcal{P}$, and $\mathcal{D}$.

Mathematical Structures

$$
\begin{aligned}
J_i^C \quad &= \quad \sum_{h,k=1}^{n} \int_{\Omega \times D_u^2 \times D_{\mathbf{v}}^2} \eta_{hk}[\mathbf{f}](\mathbf{w}_*, \mathbf{w}^*)\, \mathcal{C}_{hk}^i[\mathbf{f}](\mathbf{v}_* \to \mathbf{v},\, u_* \to u | \mathbf{w}_*, \mathbf{w}^*, u_*) \\
&\times \quad f_h(t, \mathbf{x}, \mathbf{v}_*, u_*) f_k(t, \mathbf{x}^*, \mathbf{v}^*, u^*)\, d\mathbf{v}_*\, d\mathbf{v}^*\, du_*\, du^*\, d\mathbf{x}^*,
\end{aligned}
$$

$$
J_i^L = \sum_{k=1}^{n} f_i(t, \mathbf{x}, \mathbf{v}) \int_{\Omega \times D_u \times D_{\mathbf{v}}} \eta_{ik}[\mathbf{f}](\mathbf{w}_*, \mathbf{w}^*)\, f_k(t, \mathbf{x}^*, \mathbf{v}^*, u^*)\, d\mathbf{v}^*\, du^*\, d\mathbf{x}^*,
$$

$$
\begin{aligned}
J_i^P \quad &= \quad \sum_{h,k=1}^{n} \int_{\Omega \times D_u^2 \times D_{\mathbf{v}}} \mu_{hk}[\mathbf{f}](\mathbf{w}_*, \mathbf{w}^*)\, \mathcal{P}_{hk}^i[\mathbf{f}](u_*, u^*) \\
&\times \quad f_h(t, \mathbf{x}, \mathbf{v}, u_*) f_k(t, \mathbf{x}^*, \mathbf{v}^*, u^*)\, d\mathbf{v}^*\, du_*\, du^*\, d\mathbf{x}^*.
\end{aligned}
$$

$$
\begin{aligned}
J_i^D \quad &= \quad \sum_{k=1}^{n} f_i(t, \mathbf{x}, \mathbf{v}) \int_{\Omega \times D_u \times D_{\mathbf{v}}} \mu_{ij}[\mathbf{f}](\mathbf{w}_*, \mathbf{w}^*)\, \mathcal{D}_{ij}[\mathbf{f}](u_*, u^*) \\
&\times \quad f_k(t, \mathbf{x}^*, \mathbf{v}^*, u^*)\, d\mathbf{v}^*\, du^*\, d\mathbf{x}^*.
\end{aligned}
$$

1 - Towards a Mathematical Theory of Living Systems

Mathematical Structures: Vanishing mechanical variables

$$
\begin{aligned}
\partial_t f_i(t,u) &= \left[\mathcal{C}_i[\mathbf{f}] + \mathcal{P}_i[\mathbf{f}] - \mathcal{L}_i[\mathbf{f}] - \mathcal{D}_i[\mathbf{f}]\right](t,u) \\
&= \sum_{h,k=1}^{n} \int_{D_u \times D_u} \eta_{hk}[\mathbf{f}](u_*,u^*) \mathcal{C}_{hk}^i[\mathbf{f}]\,(u_* \to u | u_*, u^*)\, f_h(t,u^*) f_k(t,u^*)\, du_*\, du^* \\
&+ \sum_{h,k=1}^{n} \int_{D_u} \int_{D_u} \mu_{hk}[\mathbf{f}](u_*,u^*)\, \mathcal{P}_{hk}^i[\mathbf{f}](u_*,u^*) f_h(t,u^*) f_k(t,u^*)\, du_*\, du^* \\
&\quad - f_i(t,u) \sum_{k=1}^{n} \int_{D_u} \eta_{ik}[\mathbf{f}](u,u^*)\, f_k(t,u^*)\, du^* \\
&\quad - f_i(t,u) \sum_{k=1}^{n} \int_{D_u} \mu_{ik}[\mathbf{f}](u,u^*)\, \mathcal{D}_{ik}[\mathbf{f}]\, f_k(t,u^*)\, du^*,
\end{aligned}
$$

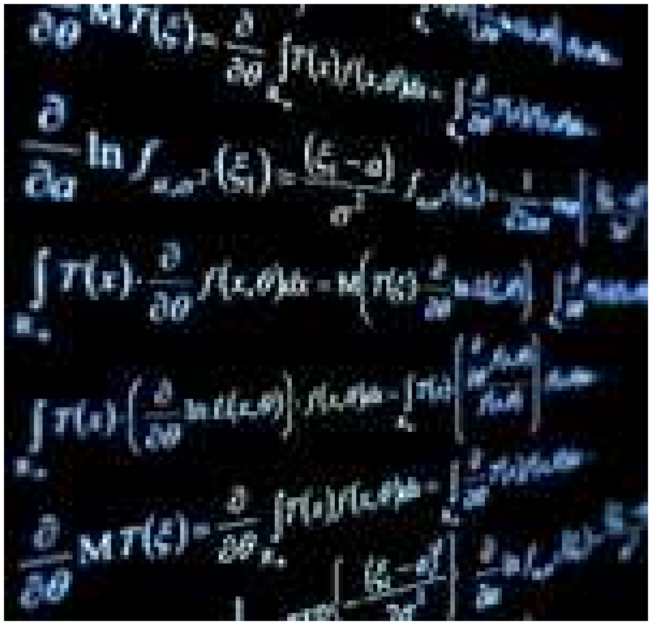
Detailed analysis of nonlinearities

N.B., V. COSCIA Nonlinearity in the Kinetic Theory for Active Particles with Focus on the Formation of Political Opinions , AMS Series in Contemporary Mathematics, 594, (2013), 99-113.

2 - Social Dynamics Looking for the Black Swan

1. Towards a Mathematical Theory of Living Systems

2. Social Dynamics Looking for the Black Swan



3. Social Behaviors in Crowds

2 - Social Dynamics Looking for the Black Swan

Looking for the Black Swan in Social Dynamics

• The dynamics of social and economic systems are necessarily based on individual behaviors, by which single subjects express, either consciously or unconsciously, a particular strategy, which is heterogeneously distributed..

• A radical philosophical change has been undertaken in social and economic disciplines. An interplay among Economics, Psychology, and Sociology has taken place, thanks to a new cognitive approach no longer grounded on the traditional assumption of rational socio-economic behavior. Starting from the concept of bounded rationality, the idea of Economics as a subject highly affected by individual (rational or irrational) behaviors, reactions, and interactions has begun to impose itself.

• A key experimental feature of such systems is that interaction among heterogeneous individuals often produces unexpected outcomes, which were absent at the individual level, and are commonly termed emergent behaviors.

• **Mathematical models should also focus, in particular, on the prediction of the so called *Black Swan***. The latter is defined to be a rare event, showing up as an irrational collective trend generated by possibly rational individual behaviors.

2 - Social Dynamics Looking for the Black Swan

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2 - Social Dynamics Looking for the Black Swan

Complexity Features

- Living $\rightarrow$ active entities
- Behavioral strategies, bounded rationality $\rightarrow$ randomness of human behaviors
- Heterogeneous distribution of strategies $\rightarrow$ stochastic games
- Evolutive dynamics $\rightarrow$ Behavioral strategies can change in time
- Collective swarm intelligence $\rightarrow$ Self-organized collective behavior can emerge spontaneously: In particular the so-called Black Swan.

Classification and Structures

- Social classes: (poor) $u_1 = -1, \dots, u_i, \dots, u_n = 1$ (wealthy)
- Political opinion: (dissensus) $v_1 = -1, \dots, v_r, \dots, v_m = 1$ (consensus)
- Distribution function: $f_i^r(t) =$ # people in u_i with opinion v_r at time t

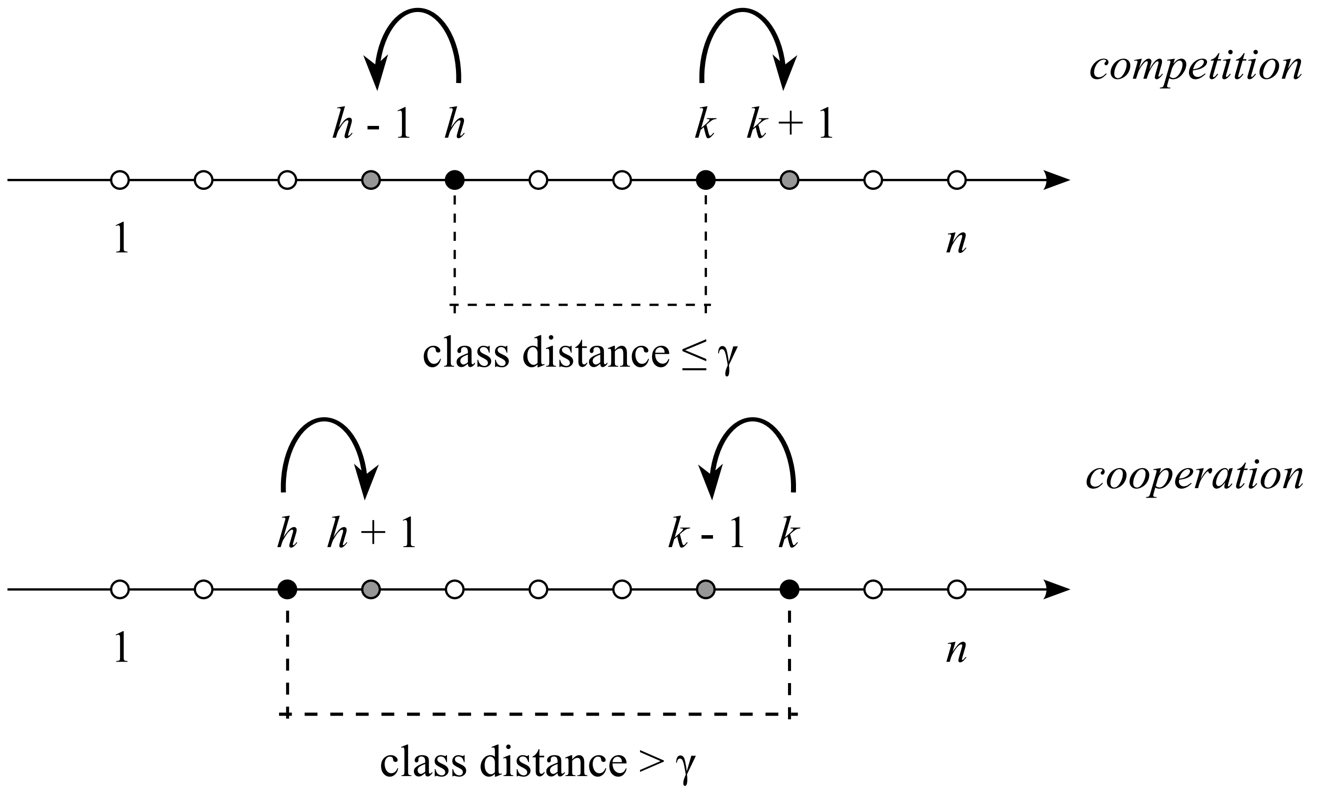
$$\frac{df_i^r}{dt} = \underbrace{\sum_{p,\,q=1}^{m} \sum_{h,\,k=1}^{n} \eta_{hk}^{pq} A_{hk}^{pq}(i,\,r) f_h^p f_k^q}_{\text{Gain}} - \underbrace{f_i^r \sum_{q=1}^{m} \sum_{k=1}^{n} \eta_{ik}^{rq} f_k^q}_{\text{Loss}}$$

$A_{hk}^{pq}(i,r) := \mathbb{P}((u_h, v_p) \to (u_i, v_r) | (u_k, v_q))$

$$\sum_{r=1}^{m} \sum_{i=1}^{n} A_{hk}^{pq}(i,\,r) = 1, \quad \forall\, h,\,k = 1, \dots, n, \quad \forall\, p,\,q = 1, \dots, m.$$

2 - Social Dynamics Looking for the Black Swan

Cooperation/Competition games



A critical distance triggers either cooperation or competition among the classes. If the distance is lower than the critical one then a competition takes place, while if it is greater than the critical one then the social organization forces cooperation.

2 - Social Dynamics Looking for the Black Swan

Modeling interactions

- *Interaction rate.* Two different rates of interaction are considered, corresponding to competitive and cooperative interactions, respectively.
- *Strategy leading to the transition probabilities.* When interacting with other particles, each active particle plays a game with stochastic output. If the difference of wealth class between the interacting particles is lower than a critical distance $\gamma[\mathbf{f}]$ (where, here and henceforth, square brackets indicate a functional dependence on the probability distribution $\mathbf{f}$) then the particles compete in such a way that those with higher wealth increase their state against those with lower wealth. Conversely, if the difference of wealth class is higher than $\gamma[\mathbf{f}]$ then the opposite occurs. The critical distance evolves in time according to the global wealth distribution over wealthy and poor particles.
- The *critical distance* $\gamma[\mathbf{f}]$ is here assumed to depend on the instantaneous distribution of the active particles over the wealth classes, such that the time evolution of $\gamma[\mathbf{f}]$ such that it grows with the number of poor active particles, thus causing larger and larger gaps of social competition.

Modeling interactions

$$S[\mathbf{f}] := N^-[\mathbf{f}] - N^+[\mathbf{f}] = \sum_{i=1}^{\frac{n-1}{2}} f_i(t) - \sum_{i=\frac{n+3}{2}}^{n} f_i(t).$$

- $S[\mathbf{f}] = S_0 \Rightarrow \gamma[\mathbf{f}] = \gamma_0$, where S_0, γ_0 are a reference social gap and the corresponding reference critical distance, respectively;
- $S[\mathbf{f}] = 1 \Rightarrow \gamma[\mathbf{f}] = n$: when the population is composed by poor particles only ($N^- = 1$, $N^+ = 0$) the socio-economic dynamics are of full competition;
- $S[\mathbf{f}] = -1 \Rightarrow \gamma[\mathbf{f}] = 0$: when the population is composed by wealthy particles only ($N^- = 0$, $N^+ = 1$) the socio-economic dynamics are of full cooperation.

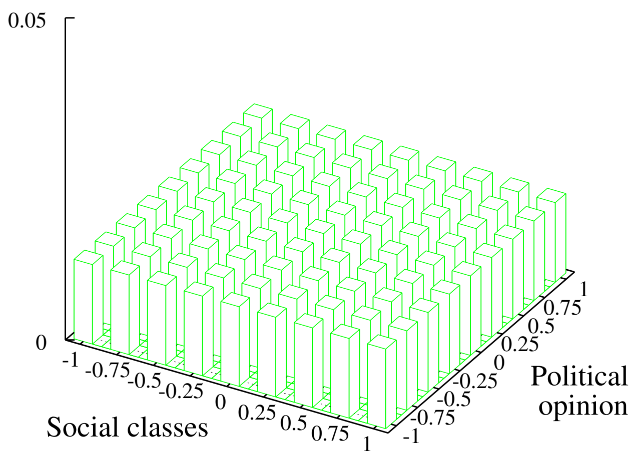
$$\gamma[\mathbf{f}] = \frac{2\gamma_0(S[\mathbf{f}]^2 - 1) - n(S_0 + 1)(S[\mathbf{f}]^2 - S_0)}{2(S_0^2 - 1)} + \frac{n}{2} S[\mathbf{f}],$$

where $\cdot$ denotes integer part (floor).

2 - Social Dynamics Looking for the Black Swan

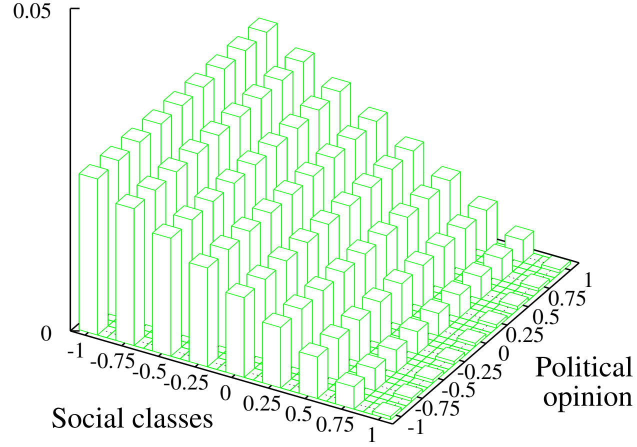
Looking for the black swan in Social Dynamics Case Studies

- Initial conditions



Society “neutral” on average

Mean wealth: 0



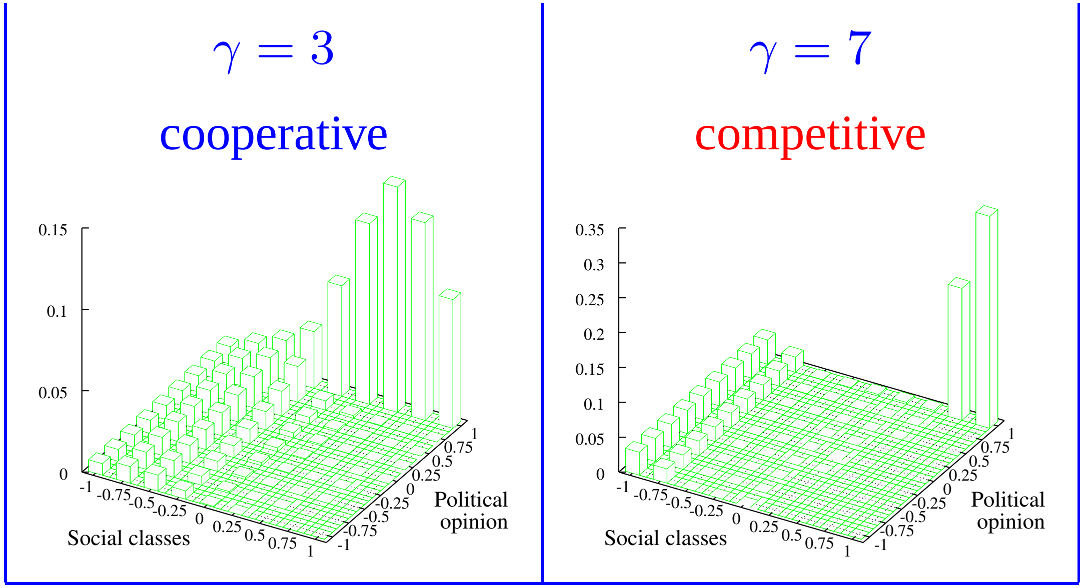
Society poor on average

Mean wealth: -0.4

2 - Social Dynamics Looking for the Black Swan

Looking for the black swan in Social Dynamics

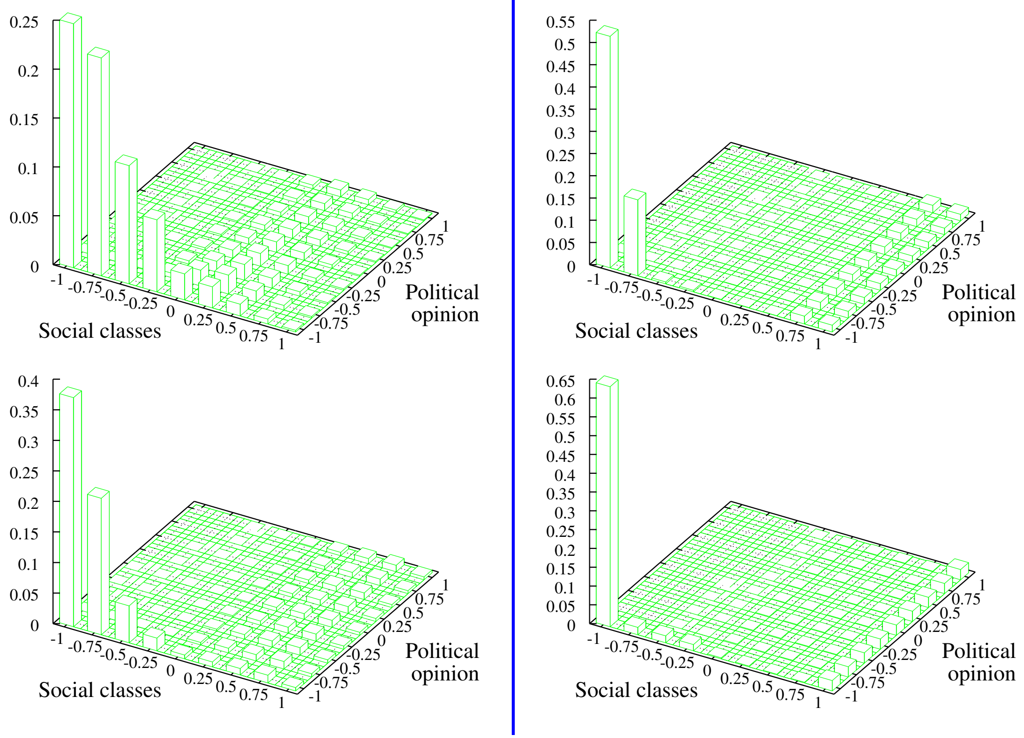
- Society which is “economically neutral” on average



Looking for the black swan in Social Dynamics

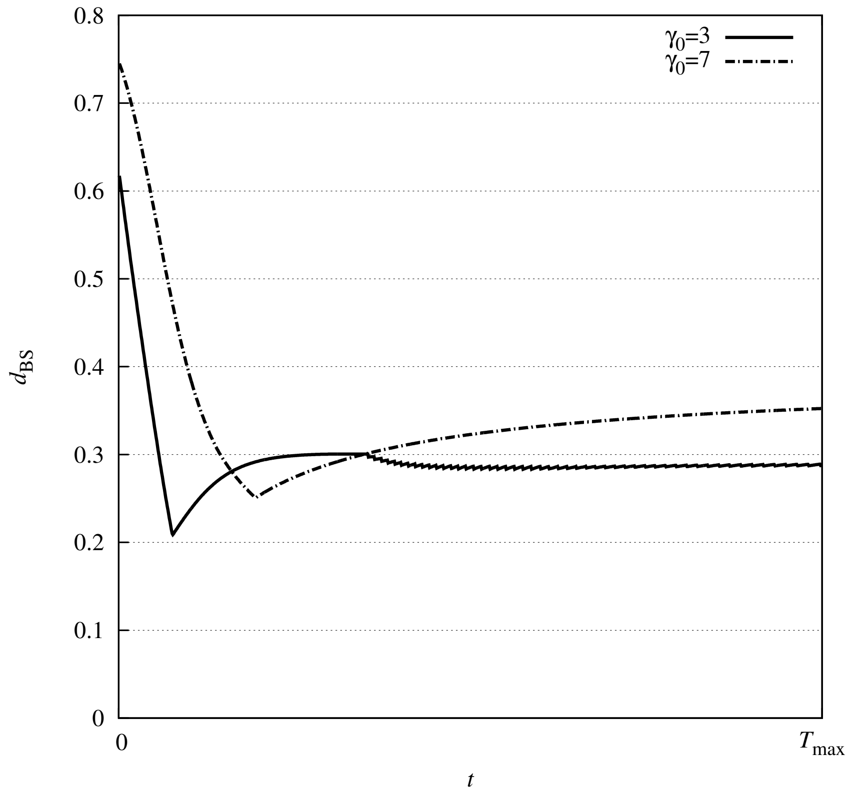
- Society which is poor on average

	$\gamma = 3$ cooperative	$\gamma = 7$ competitive
constant γ		
variable γ		



2 - Modeling Social Conflicts and Political Competition

Early Signals of the Black Swan



The mapping $t \mapsto d_{\mathrm{BS}}(t)$ *computed in the case studies with variable* γ*, taking as phenomenological guess the corresponding asymptotic distributions obtained with constant* γ.

2 - Social Dynamics Looking for the Black Swan

The role of a two scales dynamics

$$\begin{cases} \frac{df_i}{dt} = J_i[\mathbf{f};\mu] = \sum_{h,k\in I} \eta_{hk}[\mathbf{f},\mu]\,\mathcal{C}^i_{hk}[\mathbf{f},\mu] f_h f_k - f_i \sum_{k\in I} \eta_{ik}[\mathbf{f},\mu] f_k, \\ \\ \frac{d\mu}{dt} = \varepsilon\Big(\frac{1}{n}\mathbb{M}[\mathbf{f}] + \gamma\,\mathbb{N}[\mathbf{f}]\Big)\Big(1 - \frac{\mu}{2n}\Big)\mu, \end{cases} \tag{1}$$

where

$$\mathbb{N}[\mathbf{f}] := \sum_{i\in I_w} f_i(t) - \sum_{i\in I_p} f_i(t), \qquad \mathbb{M}[\mathbf{f}(t)] = \sum_{i\in I} i f_i(t).$$

2 - Social Dynamics Looking for the Black Swan

Case I – Rôle of the threshold

Low constant threshold	→	**Increasing wealth**
High constant threshold ∨ Variable threshold	→	**Decreasing wealth**

Case II - Rôle of the initial conditions

Prevalent middleclass	→	**Max. positive rate of wealth**

Case III – Interplay between initial conditions and threshold

Prevalent middleclass ∨ Prevalent wealthy cluster	∧	**Constant threshold ∨ Variable threshold**	→	**~ unchanged wealth**
Prevalent poor cluster	∧	**Constant threshold ∨ Variable threshold**	→	**different asymptotic "shapes"**

Case IV - Rôle of "selfishness"

Altruism	→	**Increasing wealth**
Selfishness	→	**Decreasing wealth**

3 - Social Behaviors in Crowds

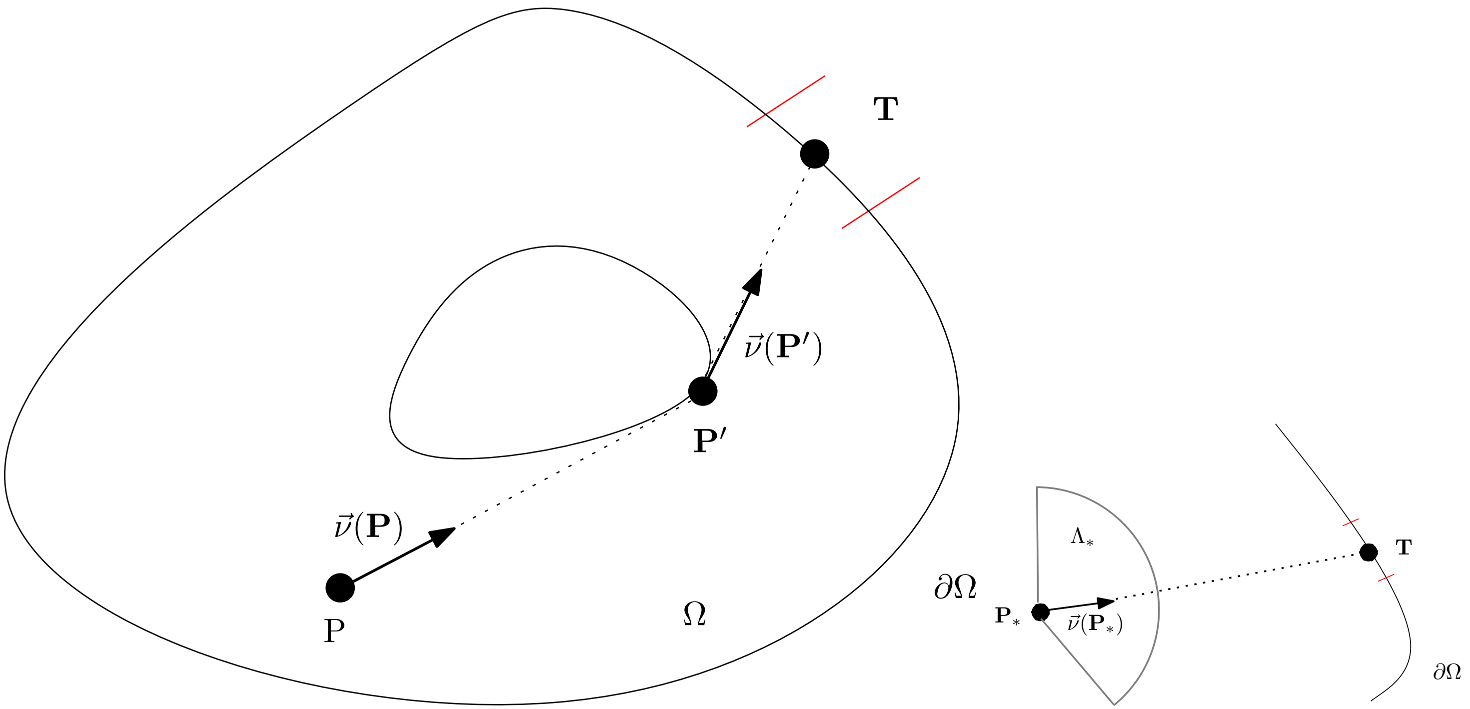
1. Towards a Mathematical Theory of Living Systems

2. Social Dynamics Looking for the Black Swan

3. Social Behaviors in Crowds

Thanks to: eVACUATE: a holistic, scenario independent, situation awareness and guidance system for sustaining the Active Evacuation Route for large crowds (Grant Agreement 313161)

Crowds in Bounded Domain with Obstacles



3 - Social Behaviors in Crowds

Active particles and micro-scale states

	Crowd dynamics
Active particles	Pedestrians
Microscopic state	Position Velocity Activity
Functional subsystems	Different abilities Individuals pursuing different targets etc.

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3. Social Behaviors in Crowds

Polar coordinates with discrete values are used for the velocity variable $\mathbf{v} = \{v, \theta\}$:

$$I_\theta = \{\theta_1 = 0\,, \ldots\,, \theta_i\,, \ldots\,, \theta_n = \frac{n}{n-1} 2\pi\}, \quad I_v = \{v_1 = 0\,, \ldots\,, v_j\,, \ldots\,, v_m = 1\}.$$

$$f(t, \mathbf{x}, \mathbf{v}, u) = \sum_{i=1}^{n} \sum_{j=1}^{m} f_{ij}(t, \mathbf{x}, u)\, \delta(\theta - \theta_i) \otimes \delta(v - v_j).$$

Some specific cases can be considered. For instance the case of two different groups, labeled with the superscript $\sigma = 1, 2$, which move towards two different targets.

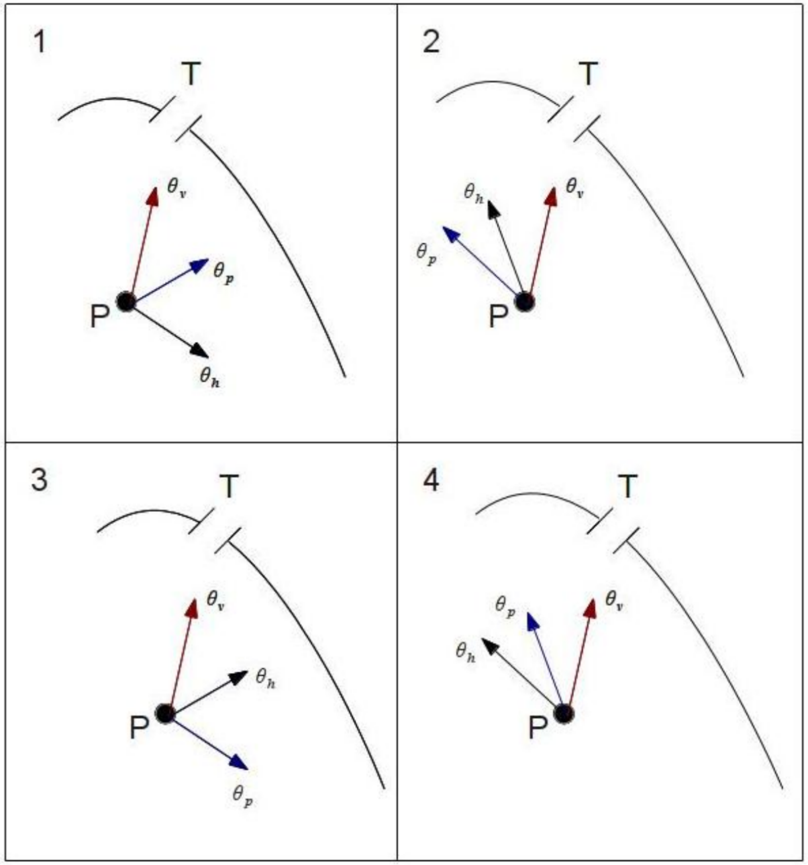
$$f^\sigma(t, \mathbf{x}, \mathbf{v}, u) = \sum_{i=1}^{n} \sum_{j=1}^{m} f_{ij}^\sigma(t, \mathbf{x})\, \delta(\theta - \theta_i) \otimes \delta(v - v_j) \otimes \delta(u - u_0)\,,$$

where $f_{ij}^\sigma(t, \mathbf{x}) = f(t, \mathbf{x}, \theta_i, v_j)$ corresponding, for each group $\sigma = 1, 2$, to the ij-particle, namely to the pedestrian moving in the direction θ_i with velocity v_j.

$$\rho(t, \mathbf{x}) = \sum_{\sigma=1}^{2} \rho^\sigma(t, \mathbf{x}) = \sum_{\sigma=1}^{2} \sum_{i=1}^{n} \sum_{j=1}^{m} f_{ij}^\sigma(t, \mathbf{x})\,,$$

3. Social Behaviors in Crowds

Interactions in the table of games



Particle in P moves to a direction θ_h (black arrow) and interacts with a field particle moving to θ_p (blue arrow), the direction to the target is θ_ν (red arrow).

3 - Social Behaviors in Crowds

$$
\begin{aligned}
\big(\partial_t &+ \mathbf{v}_{ij} \cdot \partial_{\mathbf{x}}\big) f_{ij}^{\sigma}(t, \mathbf{x}) = \mathcal{J}[\mathbf{f}](t, \mathbf{x}) \\
&= \sum_{h,p=1}^{n} \sum_{k,q=1}^{m} \int_{\Lambda} \eta[\rho(t, \mathbf{x}^*)] \mathcal{A}_{hk,pq}^{\sigma}(ij)[\rho(t, \mathbf{x}^*)] f_{hk}^{\sigma}(t, \mathbf{x})\, f_{pq}^{\sigma}(t, \mathbf{x}^*)\, d\mathbf{x}^* \\
&- f_{ij}^{\sigma}(t, \mathbf{x}) \sum_{p=1}^{n} \sum_{q=1}^{m} \int_{\Lambda} \eta[\rho(t, \mathbf{x}^*)]\, f_{pq}^{\sigma}(t, \mathbf{x}^*)\, d\mathbf{x}^*,
\end{aligned}
$$

where $\mathbf{f} = \{f_{ij}\}$, while the term $\mathcal{A}_{hk,pq}^{\sigma}(ij)$ should be consistent with the probability density property:

$$
\sum_{i=1}^{n} \sum_{j=1}^{m} \mathcal{A}_{hk,pq}^{\sigma}(ij) = 1, \quad \forall\, hp \in \{1, \ldots, n\}, \quad \forall\, kq \in \{1, \ldots, m\},
$$

for $\sigma = 1, 2$, and for all conditioning local density.

Pedestrians have a visibility zone $\Lambda = \Lambda(\mathbf{x})$, which does not coincide with the whole domain Ω due to the limited visibility angle of each individual.

3. Social Behaviors in Crowds

- **Interaction rate:**

$$\eta(\rho(t,\mathbf{x})) = \eta^0(1+\rho(t,\mathbf{x})).$$

- **Transition probability density:** The approach proposed here is based on the assumption that particles are subject to three different influences, namely the *trend to the exit point*, the *influence of the stream* induced by the other pedestrians, and the selection of the path with minimal density gradient. A simplified interpretation of the phenomenological behavior is obtained by assuming the factorization of the two probability densities modeling the modifications of the velocity direction and modulus:

$$\mathcal{A}^{\sigma}_{hk,pq}(ij) = \mathcal{B}^{\sigma}_{hp}(i)\big(\theta_h \to \theta_i | \rho(t,\mathbf{x})\big) \times \mathcal{C}^{\sigma}_{kq}(j)\big(v_k \to v_j | \rho(t,\mathbf{x})\big).$$

– *Interaction with a upper stream and target directions, namely* $\theta_p > \theta_h, \quad \theta_\nu > \theta_h$*:*

$$\begin{aligned}
\mathcal{B}^{\sigma}_{hp}(i) &= \alpha\, u_0(1-\rho) + \alpha\, u_0\, \rho \quad \text{if} \quad i = h+1\,,\\
\mathcal{B}^{\sigma}_{hp}(i) &= 1 - \alpha\, u_0(1-\rho) - \alpha\, u_0\, \rho \quad \text{if} \quad i = h\,,\\
\mathcal{B}^{\sigma}_{hp}(i) &= 0 \quad \text{if} \quad i = h-1.
\end{aligned}$$

3. Social Behaviors in Crowds

– *Interaction with a upper stream and low target direction* $\theta_p > \theta_h; \quad \theta_\nu < \theta_h$*:*

$$
\begin{aligned}
\mathcal{B}^\sigma_{hp}(i) &= \alpha\, u_0\, \rho \quad \text{if} \quad i = h+1\,,\\
\mathcal{B}^\sigma_{hp}(i) &= 1 - \alpha\, u_0(1-\rho) - \alpha\, u_0\, \rho \quad \text{if} \quad i = h\,,\\
\mathcal{B}^\sigma_{hp}(i) &= \alpha\, u_0\, (1-\rho) \quad \text{if} \quad i = h-1.
\end{aligned}
$$

– *Interaction with a lower stream and upper target direction* $\theta_p < \theta_h; \quad \theta_\nu > \theta_h$*:*

$$
\begin{aligned}
\mathcal{B}^\sigma_{hp}(i) &= \alpha\, u_0(1-\rho) \quad \text{if} \quad i = h+1\,,\\
\mathcal{B}^\sigma_{hp}(i) &= 1 - \alpha\, u_0(1-\rho) - \alpha\, u_0\, \rho \quad \text{if} \quad i = h\,,\\
\mathcal{B}^\sigma_{hp}(i) &= \alpha\, u_0\, \rho \quad \text{if} \quad i = h-1.
\end{aligned}
$$

– *Interaction with a lower stream and target directions* $\theta_p < \theta_h; \quad \theta_\nu < \theta_h$*:*

$$
\begin{aligned}
\mathcal{B}^\sigma_{hp}(i) &= 0 \quad \text{if} \quad i = h+1\,,\\
\mathcal{B}^\sigma_{hp}(i) &= 1 - \alpha\, u_0(1-\rho) - \alpha\, u_0\, \rho \quad \text{if} \quad i = h\,,\\
\mathcal{B}^\sigma_{hp}(i) &= \alpha\, u_0\, (1-\rho) + \alpha\, u_0\, \rho \quad \text{if} \quad i = h-1.
\end{aligned}
$$

3. Social Behaviors in Crowds

– *Interaction with faster particles* $v_k < v_q$ *and slower particles* $v_k > v_q$

$$\mathcal{C}^{\sigma}_{kq}(j) = \begin{cases} 1-\beta\, u_0\rho, & j=k; \\ \beta u_0\rho, & j=k+1; \\ 0, & \text{otherwise.} \end{cases} \qquad \mathcal{C}^{\sigma}_{kq}(j) = \begin{cases} \beta\, u_0\rho, & j=k; \\ 1-\beta\, u_0\rho, & j=k-1; \\ 0, & \text{otherwise.} \end{cases}$$

– *Interaction with equal velocity particles* $v_k = v_q$

$$\mathcal{C}^{\sigma}_{kq}(j) = \begin{cases} 1-2\beta\, u_0\rho, & j=k; \\ \beta\, u_0\rho, & j=k-1; \\ \beta\, u_0\rho, & j=k+1. \end{cases}$$

– *for* $k=1$ *the candidate particle cannot reduce velocity, while for* $k=k$ *cannot increase it*:

$$\mathcal{C}^{\sigma}_{kq}(j) = \begin{cases} 1-\beta\, u_0\, \rho, & j=1; \\ \beta\, u_0\, \rho, & j=2; \\ 0, & \text{otherwise;} \end{cases} \qquad \mathcal{C}^{\sigma}_{kq}(j) = \begin{cases} \beta\, u_0\, \rho, & j=m-1; \\ 1-\beta\, u_0\, \rho, & j=m; \\ 0, & \text{otherwise.} \end{cases}$$

Existence Theory

THEOREM: Let $\phi^\sigma_{ij} \in L_\infty \cap L^1$, $\phi^\sigma_{ij} \geq 0$, then there exists ϕ^0 so that, if $\| \phi \|_1 \leq \phi^0$, there exist T, a_0, and R so that a unique non-negative solution to the initial value problem exists and satisfies:

$$f \in X_T, \qquad \sup_{t \in [0,T]} \| f(t) \|_1 \leq a_0 \| \phi \|_1,$$

$$\rho(t, \mathbf{x}) \leq R, \quad \forall t \in [0,T], \quad \mathbf{x} \in \Omega.$$

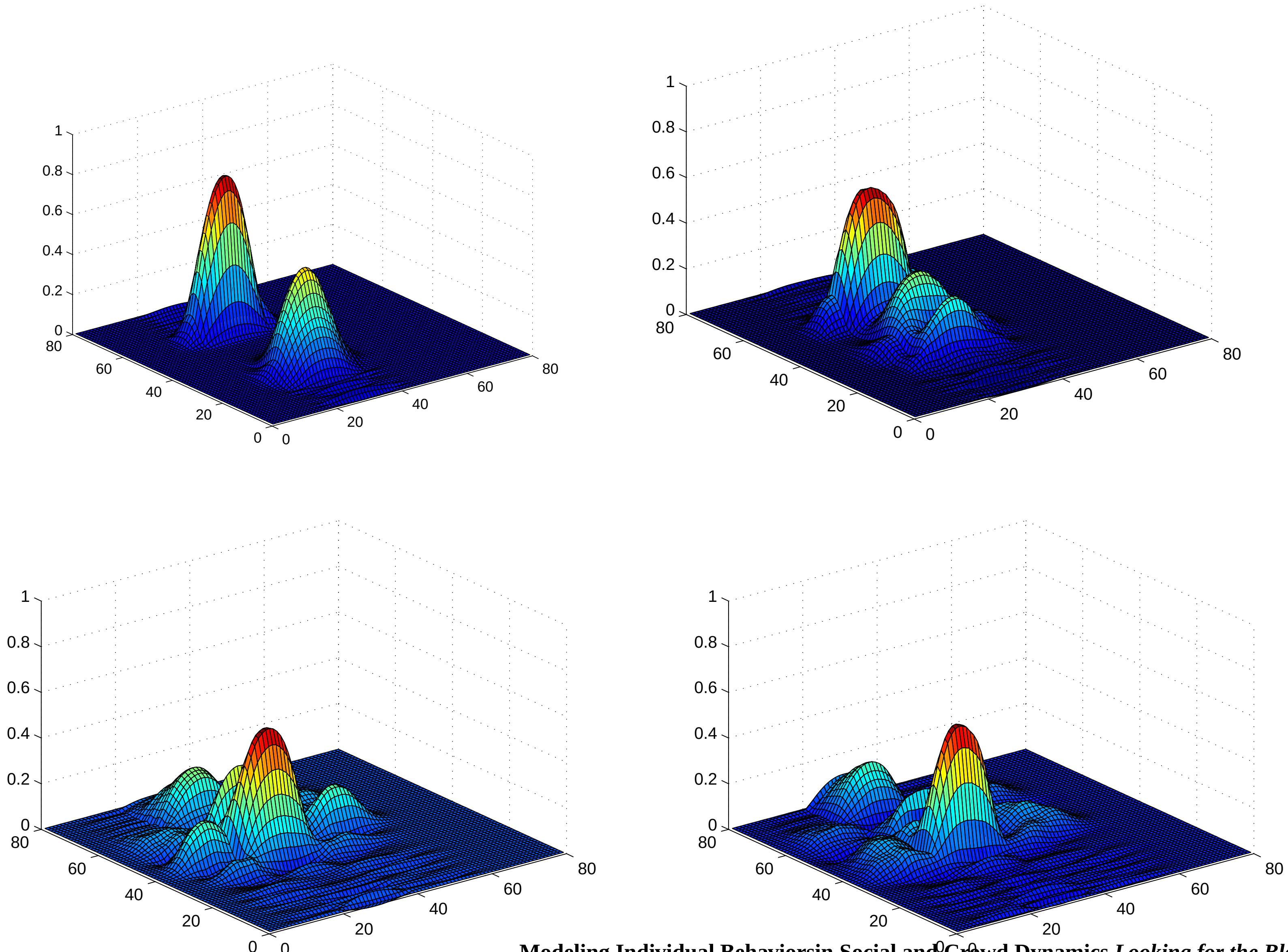
Moreover, if $\sum_{\sigma=1}^{2} \sum_{i=1}^{n} \sum_{j=1}^{m} \| \phi^\sigma_{ij} \|_\infty \leq 1$, and $\| \phi \|_1$ is small, one has $\rho(t, \mathbf{x}) \leq 1, \quad \forall t \in [0,T], \quad \mathbf{x} \in \Omega.$
There exist ϕ^r, $(r = 1, ..., p-1)$ such that if $\| \phi \|_1 \leq \phi^r$, there exists a_r so that it is possible to find a unique non-negative solution to the initial value problem satisfying for any $r \leq p-1$ the following $f(t) \in X[0, (p-1)T]$,

$$\sup_{t \in [0,T]} \| f(t + (r-1)T) \|_1 \leq a_{r-1} \| \phi \|_1,$$

and $\rho(t + (r-1)T, \mathbf{x}) \leq R, \quad \forall t \in [0,T], \quad \mathbf{x} \in \Omega$. Moreover, $\rho(t + (r-1)T, \mathbf{x}) \leq 1, \quad \forall t \in [0,T], \quad \mathbf{x} \in \Omega.$

3. Social Behaviors in Crowds

A Case Study



3. Social Behaviors in Crowds

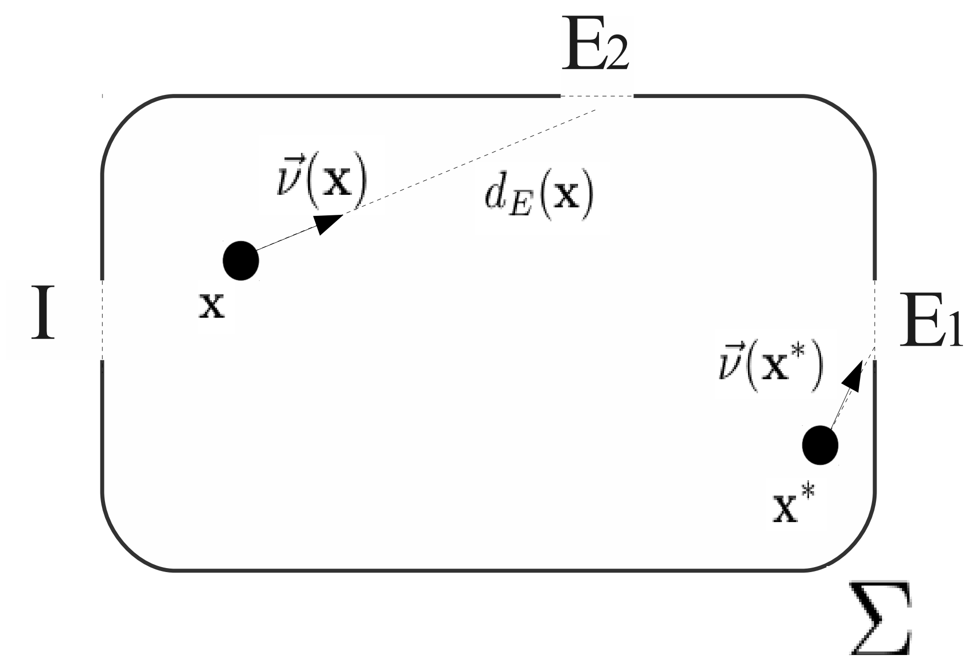
An approach to bounded domains

1. Trend to move toward the exit.
2. Trend to avoid the collision with walls.
3. Tendency to avoid congested areas.
4. Tendency to follow the stream.

3. Social Behaviors in Crowds

Trend to move toward the exit.

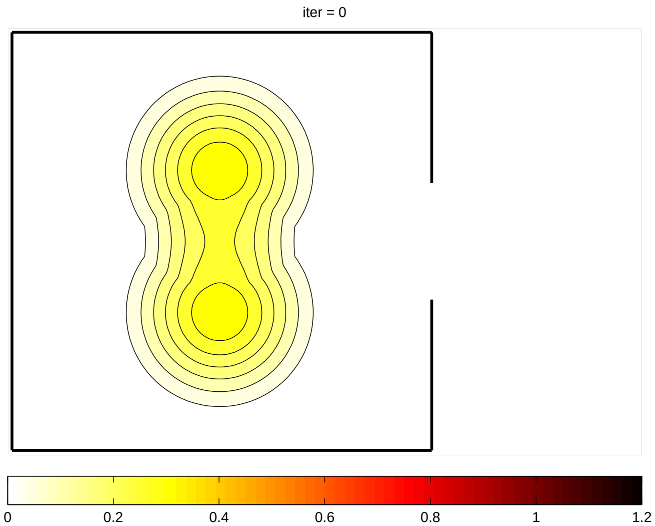
- $d_E(\mathbf{x}) =$ distance from $\mathbf{x}$ to the exit E.
- $\vec{\nu}(\mathbf{x}) =$ unitary vector pointing from $\mathbf{x}$ to E.



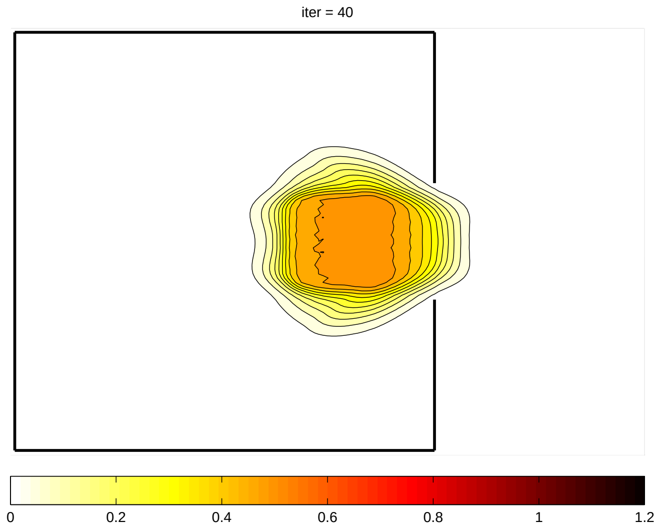
3. Social Behaviors in Crowds

Effect	**Versor**	**Geometry**	**Density**
exit	$\vec{\nu}(\mathbf{x})$	$1 - d_E(\mathbf{x})$	$1 - \rho(t, \mathbf{x})$
wall	$\vec{\tau}(\mathbf{x}, \theta_h)$	$1 - d_W(\mathbf{x}, \theta_h)$	$1 - \rho(t, \mathbf{x})$
vacuum	$\vec{\gamma}(\mathbf{x}; \rho)$	$d_E(\mathbf{x})$	$\rho(t, \mathbf{x})$
stream	$\vec{\sigma}_k$	—-	$\rho(t, \mathbf{x})$

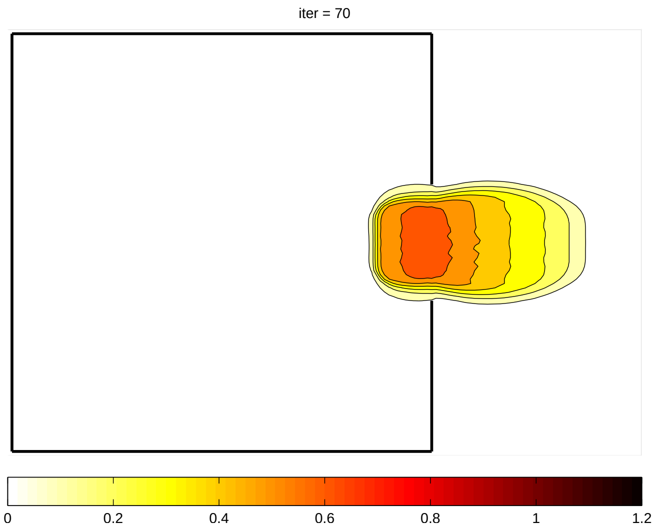
3. Social Behaviors in Crowds



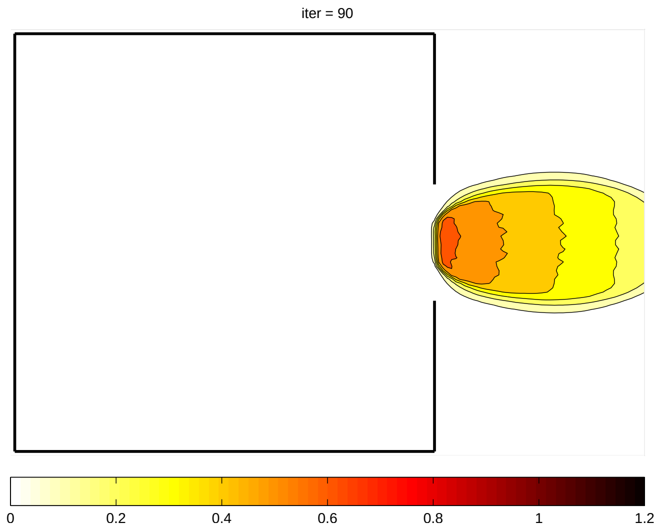
(a)



(b)



(c)



(d)

3. Social Behaviors in Crowds

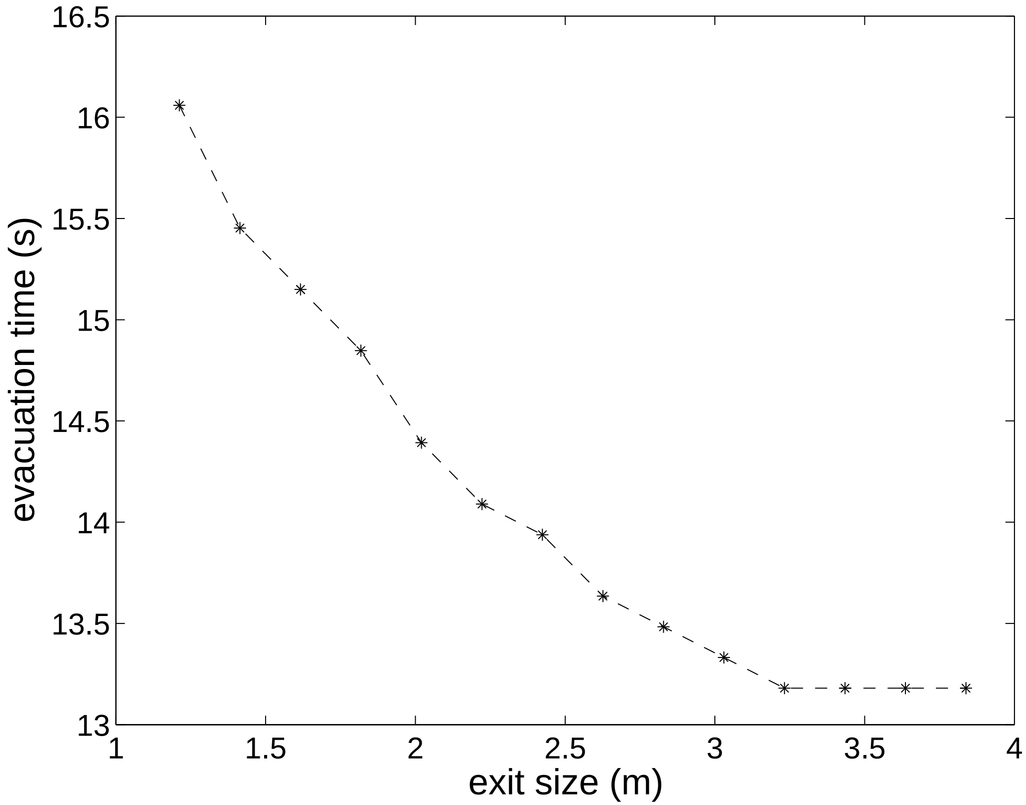
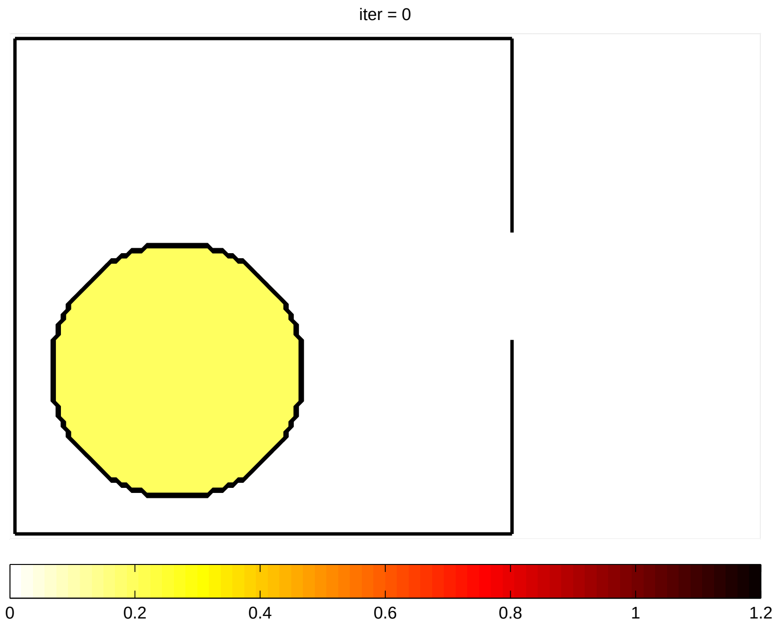
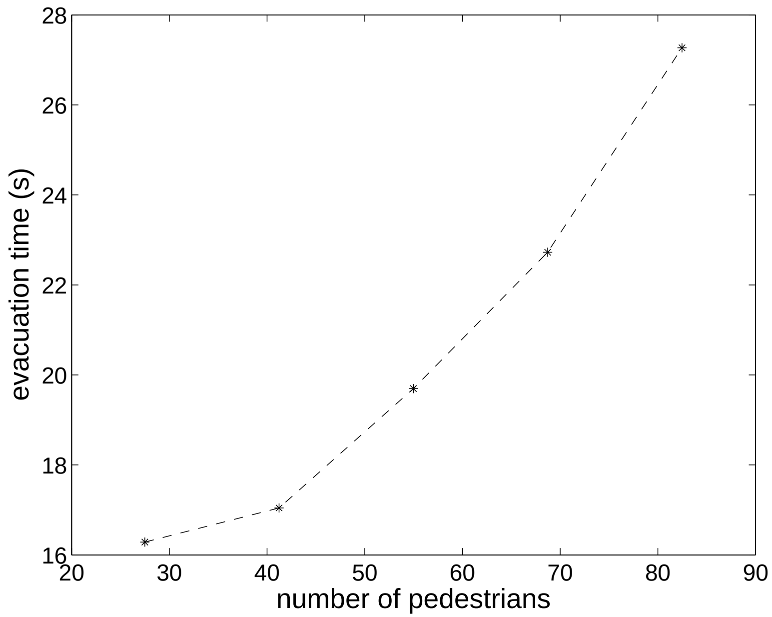


Figure 1: Case-study 1. Evacuation times for different sizes of the exit door.

3. Social Behaviors in Crowds



(a)



(b)

Figure 2: Case-study 2. (a) Pedestrians are initially distributed in a circular shape crowd with constant density. (b) Evacuation times for different initial densities.