Stochastic Differential Games and Kinetic Theory Toward the Modeling of Behavioral Social Crowds

Nicola Bellomo and Livio Gibelli
http://staff.polito.it/nicola.bellomo

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Part 1

Part 1. From the Question "What is a Crowd?" to Modeling Strategy: *Why a crowd is a social, hence "complex", system and how behaves in extreme situations such as panic?*

Part 2. The Kinetic Theory Approach to Crowd Modeling

Part 3. Validation and Analytic Challenges

1.2 - From “What is a Crowd?” to a Modeling Strategy

Bibliography of a personal quest

• N. Bellomo and C. Dogbè, On the modelling of traffic and crowds - a survey of models, speculations, and perspectives, *SIAM Review*, **53** (2011), 409–463.

• N. Bellomo and A. Bellouquid, On the modeling of crowd dynamics: Looking at the beautiful shapes of swarms, *Netw. Heter. Media*, **6** (2011), 383–399.

• N. Bellomo and A. Bellouquid, and D. Knopoff, From the micro-scale to collective crowd dynamics, *SIAM Multiscale Model. Simul.*, **11** (2013), 943–963.

• N. Bellomo and A. Bellouquid, On multiscale models of pedestrian crowds - From mesoscopic to macroscopic, *Comm. Math. Sci.*, **13** (2015), 1649–1664.

• N. Bellomo and L. Gibelli, Toward a mathematical theory of behavioral-social dynamics for pedestrian crowds, *Math. Models Methods Appl. Sci.*, **25** (2015), 2417–2437.

• N. Bellomo, S. Berrone, L. Gibelli, and A. Pieri, Macroscopic first order models of multicomponent human crowds with behavioral dynamics, in “Advances in Fluid Dynamics”, T. Tezduyar and Y. Basilev, Eds.(2016), to be published.

1.3 - From “What is a Crowd?” to a Modeling Strategy

Why a crowd is a “social, hence complex,” system and how the crowd behaves in extreme situations such as panic?

- **Ability to express a strategy:** Walkers are capable to develop specific strategies, which depend on their own state and on that of the entities in their surrounding environment.
- **Heterogeneity and hierarchy:** The ability to express a strategy is heterogeneously distributed and includes, in addition to different walking abilities, also different objectives and the possible presence of leaders.
- **Nonlinear interactions:** Interactions are nonlinearly additive and involve immediate neighbors, but also distant individuals.
- **Social communication and learning ability:** Walkers have the ability to learn from past experience. Therefore, their strategic ability evolves in time due to inputs received from outside induced by the tendency to adaptation.
- **Influence of environmental conditions:** The dynamics is affected by the quality of environment, including weather conditions, and the geometry of the domain.

1.4 - From “What is a Crowd?” to a Modeling Strategy

Complexity features of crowds Definitions by D. Helbing D. and A. Johansson, Pedestrian crowd and evacuation dynamics, *Enciclopedia of Complexity and System Science*, Springer, (2009), 6476–6495.

• ***Definition of crowd:*** Agglomeration of many people in the same area at the same time. The density of people is assumed to be high enough to cause continuous interactions, or reactions, with other individuals.

• ***Collective intelligence:*** Emergent functional behavior of a large number of people that results from interactions of individuals rather than from individual reasoning or global optimization. Establishment of a qualitatively new behavior through non-linear interactions of many individuals.

• ***Panic breakdown of ordered, cooperative behavior of individuals:*** Often, panic is characterized by attempted escape of many individuals from a real or perceived threat in situations of a perceived struggle for survival, which may end up in trampling or crushing.

1.5 - From "What is a Crowd?" to a Modeling Strategy

Levels of Description: Micro, Meso, Macro The first step of a modeling strategy is the assessment of the levels of description, their related mathematical structures, and the selection of those to be used.

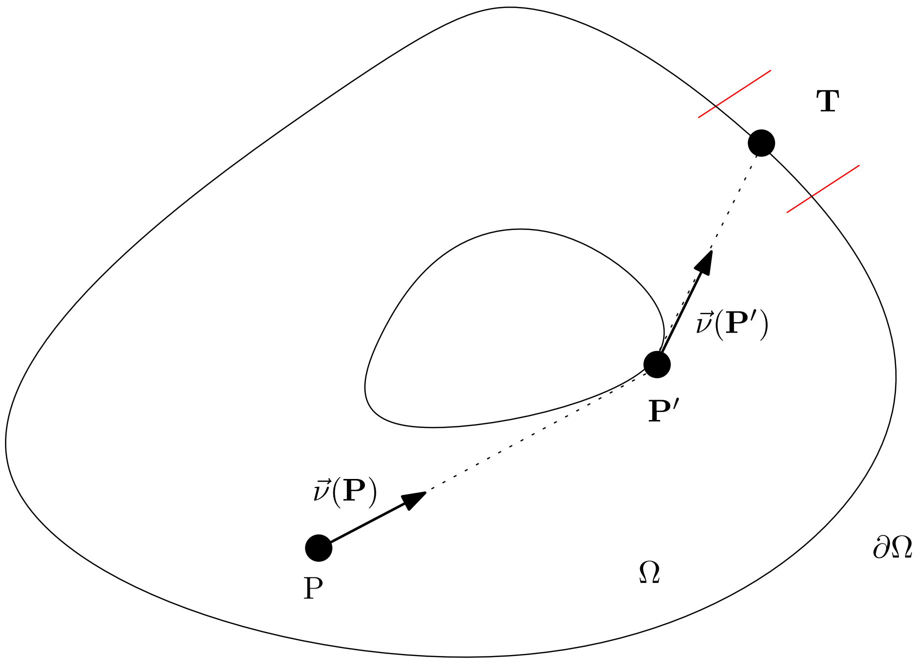
• ***Microscale:*** Walkers are individually identified. The state of the whole system is identified by their position and velocity, which dependent variables of time. Models are generally stated by systems of ordinary differential equations.

• ***Mesoscale:*** The microscopic state of the interacting entities is still identified by the position and velocity, but their representation is delivered by a suitable probability distribution function over the microscopic state. Models describe the evolution of the distribution function by nonlinear integro-differential equations.

• ***Macroscale:*** The state of the system is described by averaged gross quantities, namely density and linear momentum, regarded as dependent variables of time and space. Mathematical models describe the evolution of the above variables by systems of partial differential equations.

1.6 - From "What is a Crowd?" to a Modeling Strategy

Geometry



The set of all walls, including that of obstacles and of entrance and exit doors, is denoted by Σ.

1.7 - Scaling Problems and Mathematical Structures

Microscale: The ***microscopic description*** is represented, for each i-th walker with $i \in \{1, \ldots, N\}$, by position $\mathbf{x}_i = \mathbf{x}_i(t) = (x_i(t), y_i(t))$ and velocity $\mathbf{v}_i = \mathbf{v}_i(t) = (v_x^i(t), v_y^i(t))$.

$$\begin{cases} \dfrac{d\mathbf{x}_i}{dt} = \mathbf{v}_i \, , \\[2em] \dfrac{d\mathbf{v}_i}{dt} = \mathbf{F}_i(\mathbf{x}_1, \ldots, \mathbf{x}_N, \mathbf{v}_1, \ldots, \mathbf{v}_N; \Sigma). \end{cases}$$

where $\mathbf{F}(\cdot)$ is a psycho-mechanical acceleration acting on the i–th walker based on the action of other walkers in his/her visibility/sensitivity zone.

Remark: *The psycho-mechanical acceleration depends on* Σ, *namely on the overall geometry of the venue, as interactions take into account this feature, which is not accounted for by classical particles until these materially collide with the wall.*

Remark: *Often only binary actions are considered, that is not correct as walkers are subject to an influence domain, intersection of the visibility and sensitivity domains. Then walker develop a strategy based on all individuals within the influence domain.*

1.8 - From “What is a Crowd?” to a Modeling Strategy

- ***Macroscale:*** The ***macroscopic description*** is represented by the *local density* $\rho = \rho(t, \mathbf{x})$ and the *mean velocity* $\mathbf{V} = \mathbf{V}(t, \mathbf{x})$, which is referred to maximum mean velocity V_M of walkers.

$$\begin{cases} \partial_t \rho + \nabla_{\mathbf{x}} \cdot (\rho \mathbf{V}) = 0\,, \\ \\ \partial_t \mathbf{V} + (\mathbf{V} \cdot \nabla_{\mathbf{x}})\, \mathbf{V} = \mathcal{A}[\rho, \mathbf{V}; \Sigma]\,, \end{cases}$$

where $\mathcal{A}[\rho, \mathbf{V}; \Sigma]$ is a psycho-mechanical acceleration acting on walkers/

Remark: *The acceleration term depends on* Σ*, namely on the overall geometry of the venue, walkers take into account this feature, when they develop their walking strategy.*

Remark: *Phenomenological models are needed to describe the acceleration which acts on all individuals in the elementary volume of space due to the surrounding individuals and geometry of the venue. First order models use only the first equation closed by a phenomenological model of the type* $\mathbf{V}[\rho; \Sigma]$.

1.9 - From "What is a Crowd?" to a Modeling Strategy

Mesoscale: The system is described by the probability distribution function over the microscopic state of walkers defined by their position $\mathbf{x} \in \Sigma$ and velocity $\mathbf{v} \in D_{\mathbf{v}} \subset \mathbb{R}^2$: $f = f(t, \mathbf{x}, \mathbf{v}) =: \quad [0, T] \times \Omega \times D_{\mathbf{v}} \rightarrow \mathbb{R}_+$, such that $f(t, \mathbf{x}, \mathbf{v})\, d\mathbf{x}\, d\mathbf{v}$ denotes the number of active particles whose state, at time t, is in the interval $[\mathbf{w}, \mathbf{w} + d\mathbf{w}]$. Macroscopic quantities are obtained by velocity weighted moments. As an example local density and flux are obtained as follows:

$$\rho[f](t, \mathbf{x}) = \int_{D_{\mathbf{v}}} f(t, \mathbf{x}, \mathbf{v})\, d\mathbf{v}\,, \qquad \mathbf{q}[f](t, \mathbf{x}) = \int_{D_{\mathbf{v}}} \mathbf{v}\, f(t, \mathbf{x}, \mathbf{v})\, d\mathbf{v},$$

The dynamics is obtained by a balance of microscopic entities in the elementary volume of the space of microscopic state. This amounts to equate the transport of f to the net flow (inlet minus outlet) due to interactions. The formal result is as follows:

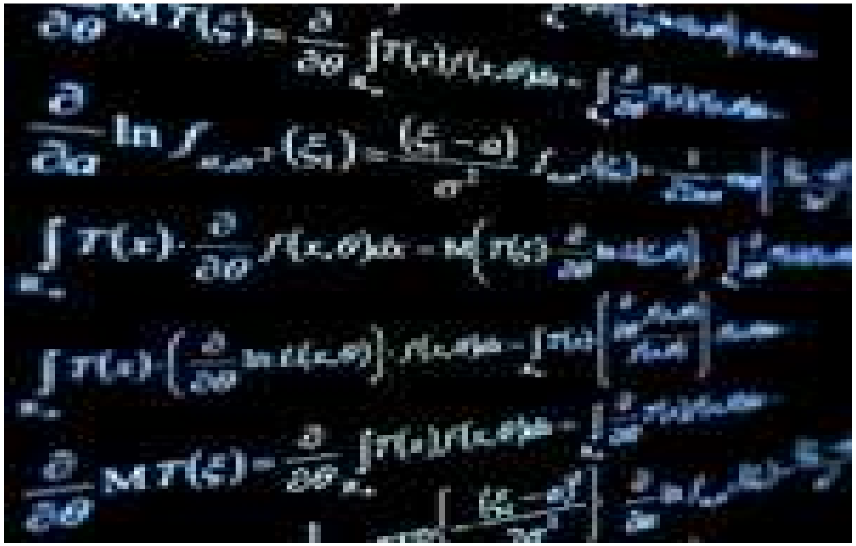
$$(\partial_t + \mathbf{v} \cdot \partial_{\mathbf{x}})\; f(t, \mathbf{x}) = \big(J^+ - J^-\big)[\mathbf{f}, \Sigma](t, \mathbf{x}, \mathbf{v}),$$

where J^+ and J^- are, respectively, the inlet and outlet fluxes induced by interaction among walkers and between them and the walls, obstacles and walls. Interactions are nonlinearly additive and nonlocal in space.

Part 2

1. From the Question "What is a Crowd?" to Modeling Strategy

Part 2. The Kinetic Theory Approach to Crowd Modeling: *How mathematical sciences can develop a strategy to model the "behavioral and social" dynamics of crowds?*



Part 3. Validation and Analytic Challenges

2.2 Tools of the Kinetic Theory of Active Particles

2. How mathematical sciences can develop a strategy to understand the "behavioral and social dynamics of crowds"?

Strategy: Modeling can be developed according to the following strategy:

1. Derivation of a general mathematical structure consistent with the complexity features of living systems (focusing on crowd dynamics);

2. Derivation of specific crowd models by inserting interactions models at the microscopic scale into the aforementioned structure;

3. Validation of models by exploiting the information delivered by empirical data;

4. Understanding the link between dynamics delivered by the kinetic theory approach and that at the macroscopic scale.

2.3. Tools of the Kinetic Theory of Active Particles

How complex systems can be defined?

N.B. H. Berestycki, F. Brezzi, and J.P. Nadal, *Mathematics and Complexity in Life and Human Sciences*, Mathematical Models and Methods in Applied Sciences, 2010.

• *The study of complex systems, namely systems of many individuals interacting in a non-linear manner, has received in recent years a remarkable increase of interest among applied mathematicians, physicists as well as researchers in various other fields as economy or social sciences.*

• *Their collective behavior is determined by the dynamics of their interactions. On the other hand, a traditional modeling of individual dynamics does not lead in a straightforward way to a mathematical description of collective emerging behaviors.*

In particular it is very difficult to understand and model these systems based on the sole description of the dynamics and interactions of a few individual entities localized in space and time.

2.4. Tools of the Kinetic Theory of Active Particles

Working on a new game theory

Evolutionary game theory provides an important conceptual contribution to the modeling of complex systems.

• J. Hofbauer and K. Sigmund, Evolutionary game dynamics, *Bull. Am. Math. Society*, **40** 479-519, (2003).

> Evolutionary game theory deals with entire populations of players, all programmed to use the same strategy (or type of behavior). Strategies with higher payoff will spread within the population (this can be achieved by learning, by copying or inheriting strategies, or even by infection. The payoffs depend on the actions of the co-players and hence on the frequencies of the strategies within the population).

Rather than dealing with players involved in the game with strategies that attempt to maximize *their own payoff*, in evolutionary game theory we have a whole population that is pursuing an *individual or collective wellbeing*. Tools of game theory must include the aforementioned features of evolution and learning dynamics.

2.5. Tools of the Kinetic Theory of Active Particles

1. Ability to express a strategy: Living entities are capable to develop specific *strategies* and *organization abilities* that depend on the state of the surrounding environment. These can be expressed without the application of any external organizing principle.

2. Heterogeneity: The ability to express a strategy is not the same for all entities: *Heterogeneity* characterizes a great part of living systems, namely, the characteristics of interacting entities can even differ from an entity to another belonging to the same structure.

3. Learning ability: Living systems receive inputs from their environments and have the ability to learn from past experience. Therefore their strategic ability and the characteristics of interactions among living entities evolve in time.

4. Nonlinear Interactions: Interactions nonlinearly additive and involve immediate neighbors, but in some cases also distant particles. Indeed, living systems have the ability to communicate and may possibly choose different observation paths

On a quest toward the search of a mathematical structure

Interactions by stochastic games: Living entities, at each interaction, *play a game* with an output that technically depends on their strategy somehow related to adaptation abilities. The output of the game generally is not deterministic and **the dynamics depends also on the overall shape of the walls including inlet and outlet doors**.

• **Test** particles of the i-th functional subsystem with microscopic state, at time t, delivered by the variable $(\mathbf{x}, \mathbf{v}, u) := \mathbf{w}$, whose distribution function is $f_i = f_i(t, \mathbf{x}, \mathbf{v}, u) = f_i(t, \mathbf{w})$. The test particle is assumed to be representative of the whole system.

• **Field** particles of the k-th functional subsystem with microscopic state, at time t, defined by the variable $(\mathbf{x}^*, \mathbf{v}^*, u^*) := \mathbf{w}^*$, whose distribution function is $f_k = f_k(t, \mathbf{x}^*, \mathbf{v}^*, u^*) = f_k(t, \mathbf{w}^*)$.

• **Candidate** particles, of the h-th functional subsystem, with microscopic state, at time t, defined by the variable $(\mathbf{x}_*, \mathbf{v}_*, u_*) := \mathbf{w}_*$, whose distribution function is $f_h = f_h(t, \mathbf{x}_*, \mathbf{v}_*, u_*) = f_h(t, \mathbf{w}_*)$.

2.7. Tools of the Kinetic Theory of Active Particles

H.1. Candidate or test particles in $\mathbf{x}$, interact with the field particles in the interaction domain $\mathbf{x}^* \in \Omega$. Interactions are weighted by the ***interaction rate*** $\eta_{hk}[\mathbf{f}]$, which is supposed to depend on the local distribution function at the position of the field particles.

H.2. A candidate particle modifies its state according to the probability density: $\mathcal{C}^i_{hk}[\mathbf{f}](\mathbf{v}_* \to \mathbf{v},\, u_* \to u|\mathbf{w}_*, \mathbf{w})$, which denotes the probability density that a candidate particles of the h-subsystems with state $\mathbf{w}_* = \{\mathbf{x}_*, \mathbf{v}_*, u_*\}$ reaches the state $\{\mathbf{v}, u\}$ in the i-th subsystem after an interaction with the field particles of the k-subsystems with state $\mathbf{w}^* = \{\mathbf{x}^*, \mathbf{v}^*, u^*\}$.

Normalized **dimensionless** variables are used by dividing the number n of people per unit area with respect to the maximum number n_M corresponding to packing, and the velocity modulus (speed) v_r to the limit velocity v_ℓ

$$\rho = \frac{n}{n_M}, \qquad v = \frac{v_r}{v_\ell}.$$

A mathematical structure

Variation rate of the number of active particles

$=$ *Inlet flux rate caused by number conservative interactions*

$-$*Outlet flux rate caused by conservative interactions*,

which corresponds to the following structure:

$$
\begin{aligned}
(\partial_t + \mathbf{v}\cdot\partial_{\mathbf{x}})\, f_i(t,\mathbf{x},\mathbf{v},u) &= \big(J_i^C - J_i^L\big)[\mathbf{f}](t,\mathbf{x},\mathbf{v},u) \\
&= \sum_{h,k=1}^{n} \int_{\Omega\times D_u^2\times D_{\mathbf{v}}^2} \eta_{hk}[\mathbf{f}](\mathbf{w}_*,\mathbf{w}^*)\, \mathcal{C}_{hk}^i[\mathbf{f}](\mathbf{v}_*\to\mathbf{v},\, u_*\to u|\mathbf{w}_*,\mathbf{w}^*,u_*) \\
&\quad \times f_h(t,\mathbf{x},\mathbf{v}_*,u_*) f_k(t,\mathbf{x}^*,\mathbf{v}^*,u^*)\, d\mathbf{v}_*\, d\mathbf{v}^*\, du_*\, du^*\, d\mathbf{x}^* \\
&\quad - \sum_{k=1}^{n} f_i(t,\mathbf{x},\mathbf{v}) \int_{\Omega\times D_u\times D_{\mathbf{v}}} \eta_{ik}[\mathbf{f}](\mathbf{w}_*,\mathbf{w}^*)\, f_k(t,\mathbf{x}^*,\mathbf{v}^*,u^*)\, d\mathbf{v}^*\, du^*\, d\mathbf{x}^*.
\end{aligned}
$$

2.9. Tools of the Kinetic Theory of Active Particles

From the mathematical structure to models

- The overall system is subdivided into *functional subsystems* constituted by entities, called *active particles*, whose individual state is called *activity*;
- Each functional subsystem is featured by different ways of expressing their own strategy;
- The state of each functional subsystem is defined by a time dependent, probability distribution over the micro-scale state, which includes position, velocity, and activity;
- Interactions are modeled by games theory, more precisely stochastic games, where the state of the interacting particles and their outputs are known in probability;
- The evolution of the probability distribution is obtained by a balance of number particles within elementary volume of the space of the microscopic states, where the dynamics of inflow and outflow of particles is related to interactions.

2.10. *Tools of the Kinetic Theory of Active Particles*

Active particles, micro-scale states, and environment

Active particles	Walkers
Microscopic state	Position Velocity Activity
Functional subsystems	Different abilities Individuals pursuing different strategies Presence of leaders
Environment	Unbounded domains Domains with obstacles and boundaries Quality of the environment

2.11. Tools of the Kinetic Theory of Active Particles

Real and perceived quantities

• The dynamics in two space dimensions is considered, while polar coordinates are used for the velocity variable, namely $\mathbf{v} = \{v, \theta\}$, where v is the velocity modulus and θ denotes the velocity direction.

• The ***perceived density*** ρ^a_θ along the direction θ:

$$\rho^a_\theta = \rho^a_\theta[\rho] = \rho + \frac{\partial_\theta \rho}{\sqrt{1 + (\partial_\theta \rho)^2}} \big[(1-\rho)\, H(\partial_\theta \rho) + \rho\, H(-\partial_\theta \rho)\big],$$

where ∂_θ denotes the derivative along the direction θ, while $H(\cdot)$ is the heaviside function $H(\cdot \geq 0) = 1$, and $H(\cdot < 0) = 0$. Therefore, positive gradients increase the perceived density up to the limit $\rho = 1$, while negative gradients decrease it down to the limit $\rho = 0$ in a way that

$$\partial_\theta \rho \to \infty \Rightarrow \rho^a \to 1\,, \quad \partial_\theta \rho = 0 \Rightarrow \rho^a = \rho\,, \quad \partial_\theta \rho \to -\infty \Rightarrow \rho^a \to 0.$$

2.12. Tools of the Kinetic Theory of Active Particles

Modeling the decision process of velocity adjustment

1. In unbounded domains four types of stimuli contribute to the modification of walking direction:

1. Desire to reach a well defined target, namely a direction or a meeting point;
2. Attraction toward the mean stream;
3. Attempt to avoid overcrowded areas;
4. The presence of walls induce a stimulus to avoid them.

2. Walkers moving from one direction to the other adapt their velocity to the new local perceived density conditions, namely they decrease speed for increasing perceived density and increase it for decreasing perceived density.

3. The dynamics is more rapid in high quality areas; moreover rapidity is heterogeneously distributed and increases for high values of the activity variable.

2.13. Tools of the Kinetic Theory of Active Particles

Dynamics at the microscopic scale in three steps:

1. direction of movement is changed depending on local density, mean velocity, and trend to the exit;
2. modulus of velocity is decreased (increased) depending on the perceived density;
3. activity variable is varied according to a social dynamics based on attraction and/or repulsion of social behaviors.

$$\boldsymbol{\nu}_i^{(p)} = \frac{\tilde{\rho}\boldsymbol{\nu}^{(v)} + (1-\tilde{\rho})\,\dfrac{\beta\boldsymbol{\nu}^{(s)} + (1-\beta)\boldsymbol{\nu}_i^{(t)}}{\left\|\beta\boldsymbol{\nu}^{(s)} + (1-\beta)\boldsymbol{\nu}_i^{(t)}\right\|}}{\left\|\tilde{\rho}\boldsymbol{\nu}^{(v)} + (1-\tilde{\rho})\,\dfrac{\beta\boldsymbol{\nu}^{(s)} + (1-\beta)\boldsymbol{\nu}_i^{(t)}}{\left\|\beta\boldsymbol{\nu}^{(s)} + (1-\beta)\boldsymbol{\nu}_i^{(t)}\right\|}\right\|},$$

where $\tilde{\rho} = \rho/\rho_{\text{MAX}}$, being ρ_{MAX} the highest admissible packing density, and

$$\boldsymbol{\nu}^{(v)} = -\frac{\nabla_{\mathbf{x}}\rho}{\|\nabla_{\mathbf{x}}\rho\|}, \qquad \boldsymbol{\nu}^{(s)} = \frac{\xi}{\|\xi\|}$$

Part 3

Part 1. From the Question "What is a Crowd?" to Modeling Strategy:

Part 2. The Kinetic Theory Approach to Crowd Modeling and Validation:

Part 3. Validation and Analytic Challenges: *Does the study of crowd generate challenging analytic and computational problems?*

3.2. Validation and Analytic Challenges

How crowd models can be validated?

In general, a model is considered valid if it is mathematically well posed and its numerical implementation provides feasible and consistent results in agreement with empirical data. More specifically, the following main features need to be assessed:

1. ***Ability to capture the complexity features of the crowd viewed as a living, hence complex, system.***

2. ***Ability to reproduce empirical data in steady flow conditions.***

3. ***Ability to reproduce, at a qualitative level, emerging collective behaviors.***

Moreover, some applications ask to models to provide numerical solutions with a reduced computational time. This requirement has to be specifically referred to specific applications such as evacuation through complex venues.

3.3. Validation and Analytic Challenges

Classification of empirical data

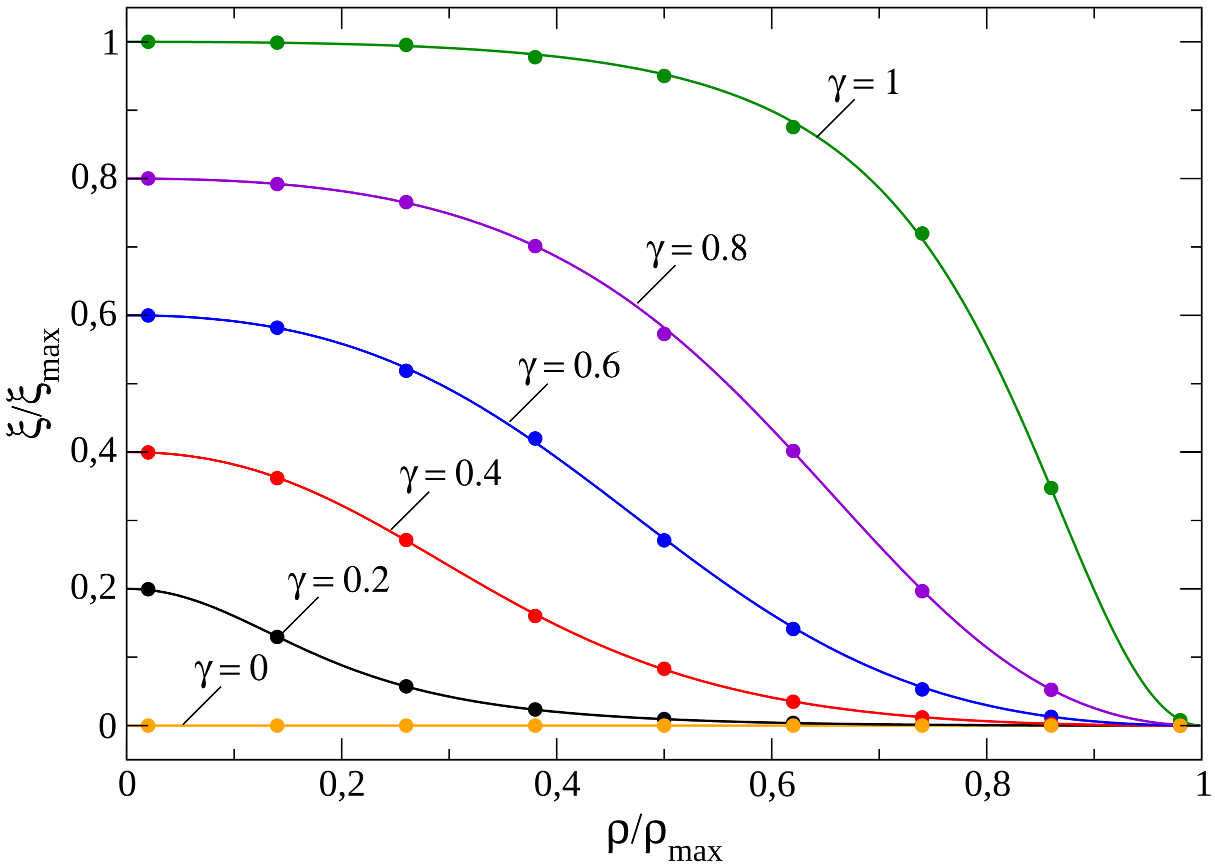
Steady flow data: The so called velocity and fundamental diagrams which plot, respectively, the average speed and flux versus the local density in steady conditions. The information provided by these diagrams is restricted to the modulus of the velocity vector since directional information is lost.

Data on emerging behaviors: Data on emerging collective behaviors provide a qualitative description of collective dynamics. Some of them are repetitively reproduced under similar physical conditions. These behaviors are subject to large quantitative deviations corresponding to small deviations of the data of the system.

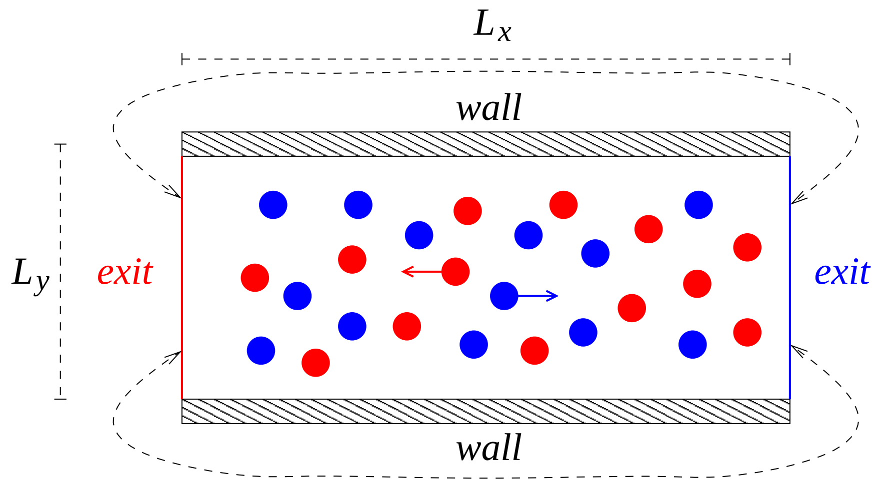
Data on individual behaviors and interactions: These data aim at understanding how pedestrians individually react to other pedestrians and how such interactions determine collective dynamics. In other words, such data help to understand whether and how local interactions can have non-local effects. Special interest is aroused by emerging behaviors in challenging evacuation dynamics.

3.4. Validation and Analytic Challenges

Fundam



Individuals walking in a corridor with opposite directions

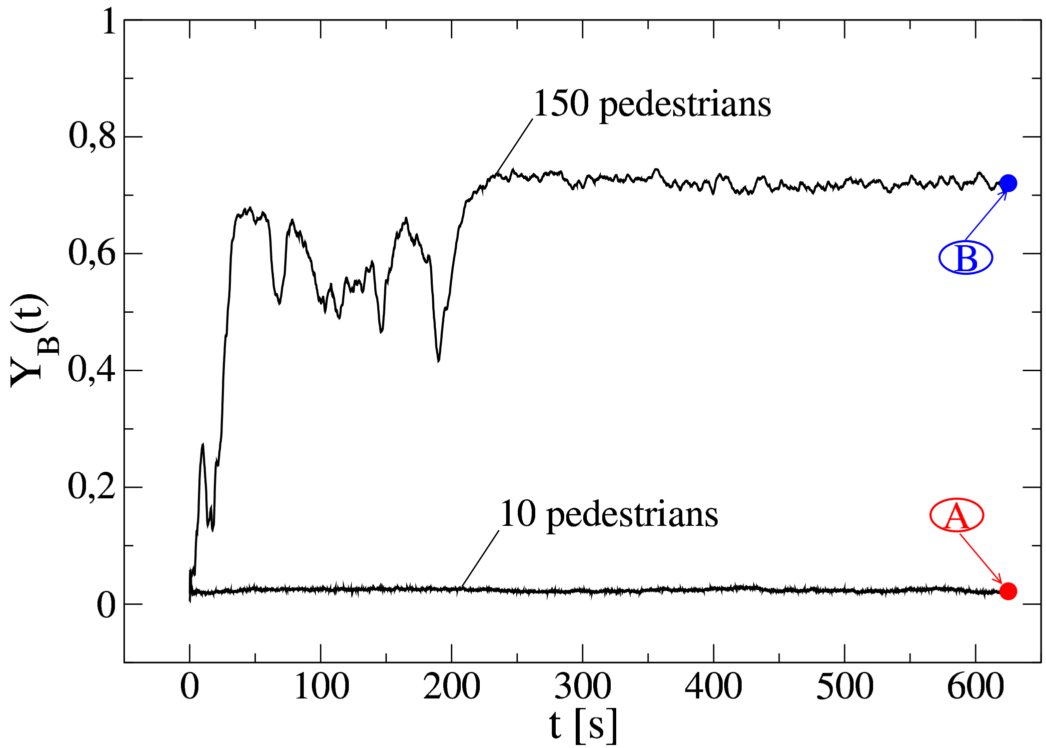


- The kinetic model of pedestrian crowds is applied to the problem of two groups of people walking in opposite directions.
- The segregation of walkers into lanes of uniform walking direction is quantitatively assess by computing the band index

$$Y_{\mathrm{B}}(t) = \frac{1}{L_x L_y} \int_0^{L_y} \left| \int_0^{L_x} \frac{\rho_1(t,\mathbf{x}) - \rho_2(t,\mathbf{x})}{\rho_1(t,\mathbf{x}) + \rho_2(t,\mathbf{x})} dx \right| dy$$

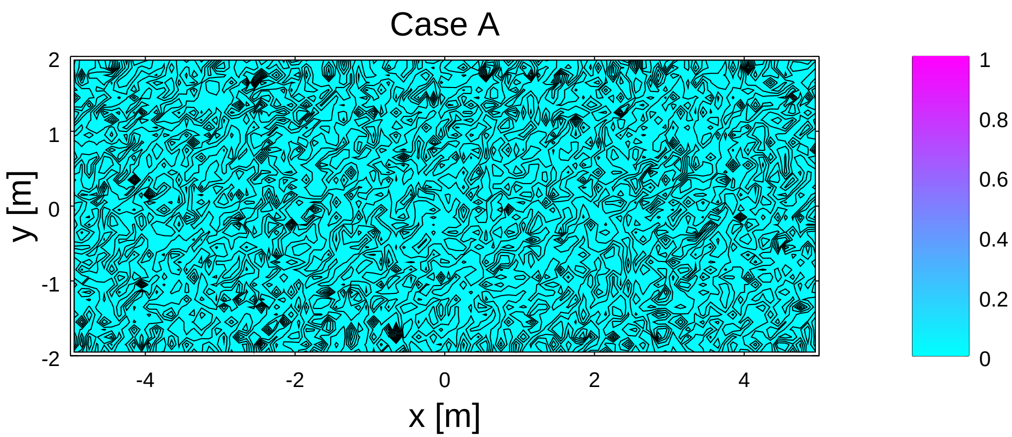
3.6. Validation and Analytic Challenges

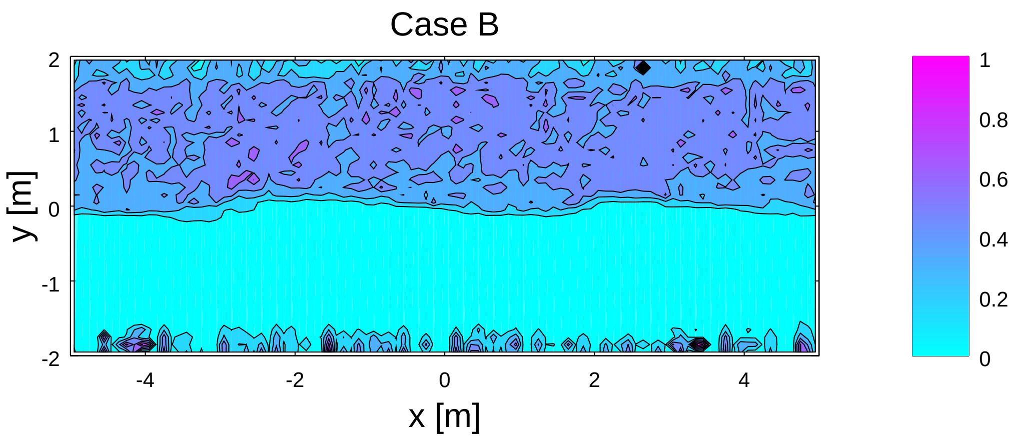
Pedestrians walking in a corridor with opposite directions



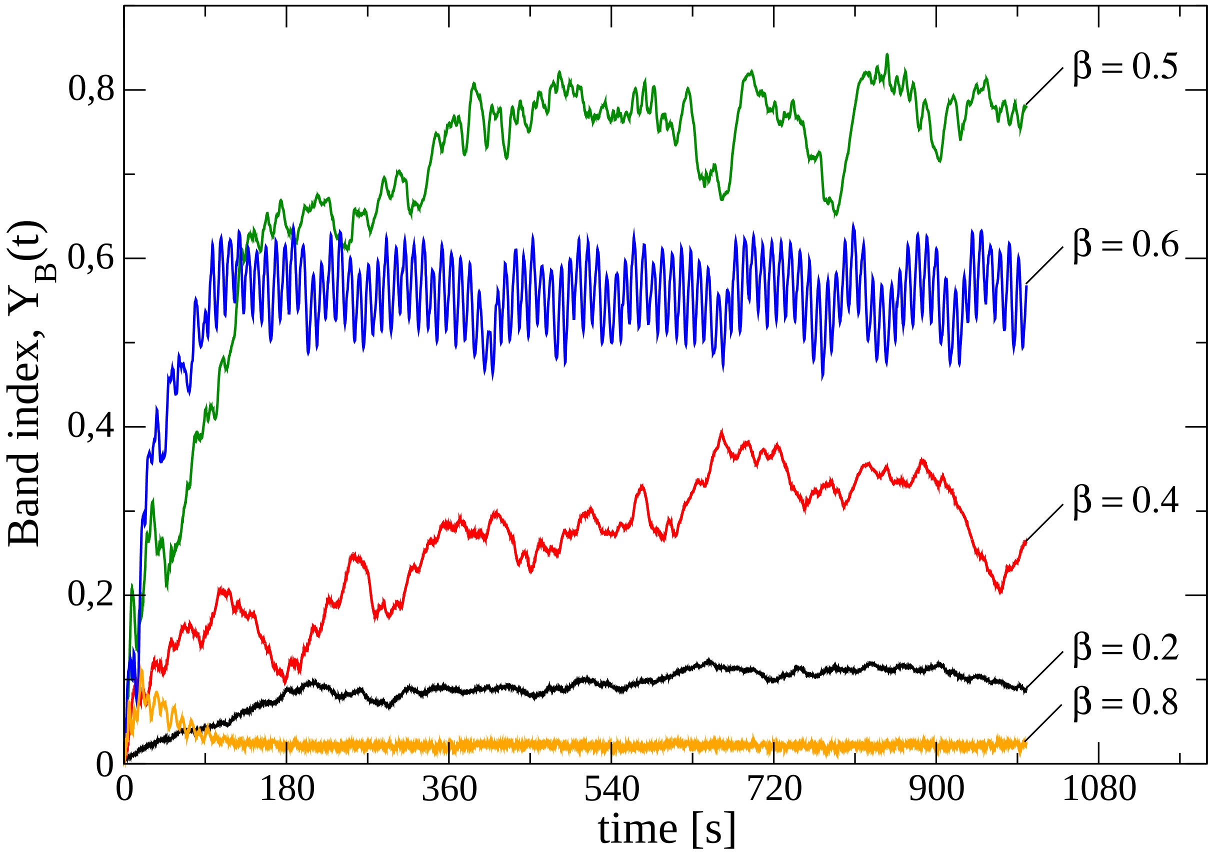
3.7. Validation and Analytic Challenges

A: Low density flow; B: high density flow; $\varepsilon = .8$

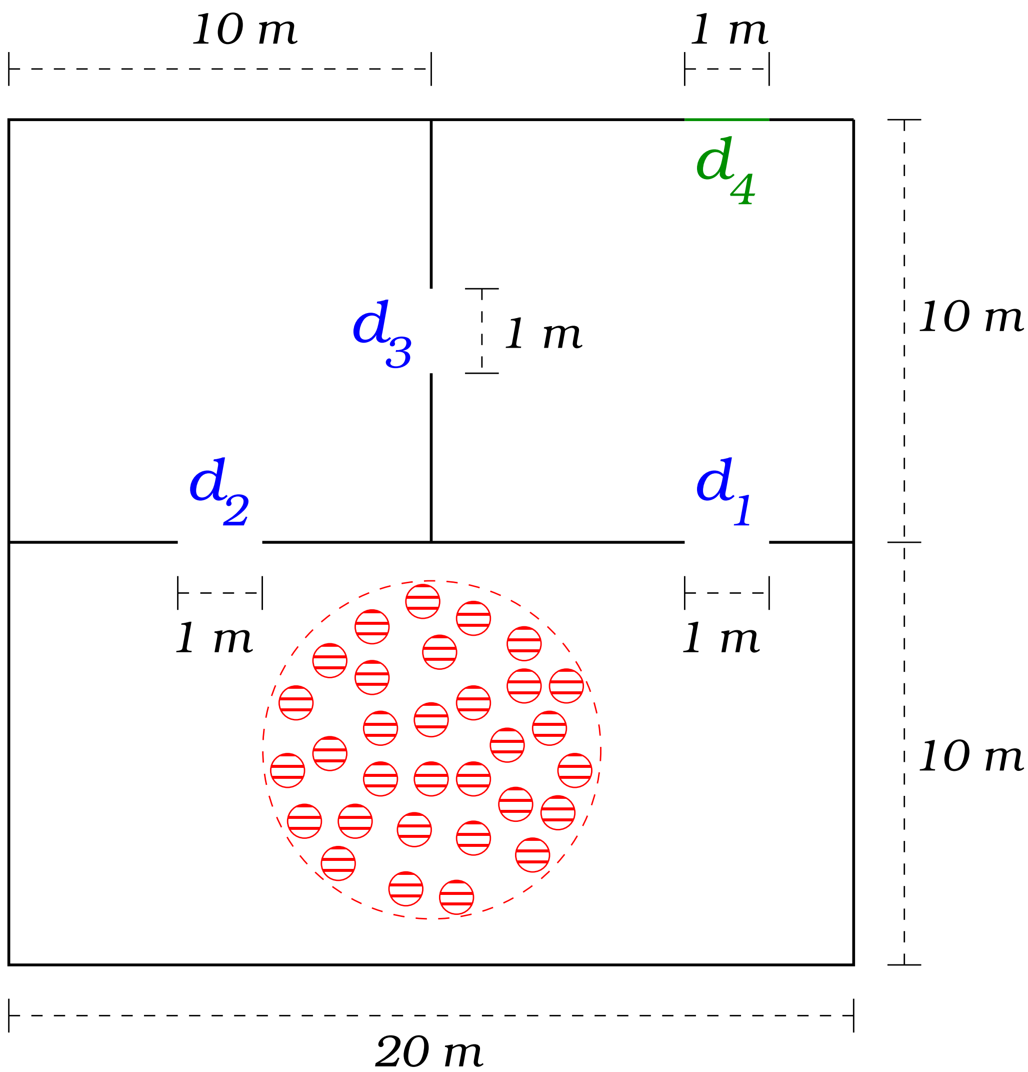




3.8. Validation and Analytic Challenges

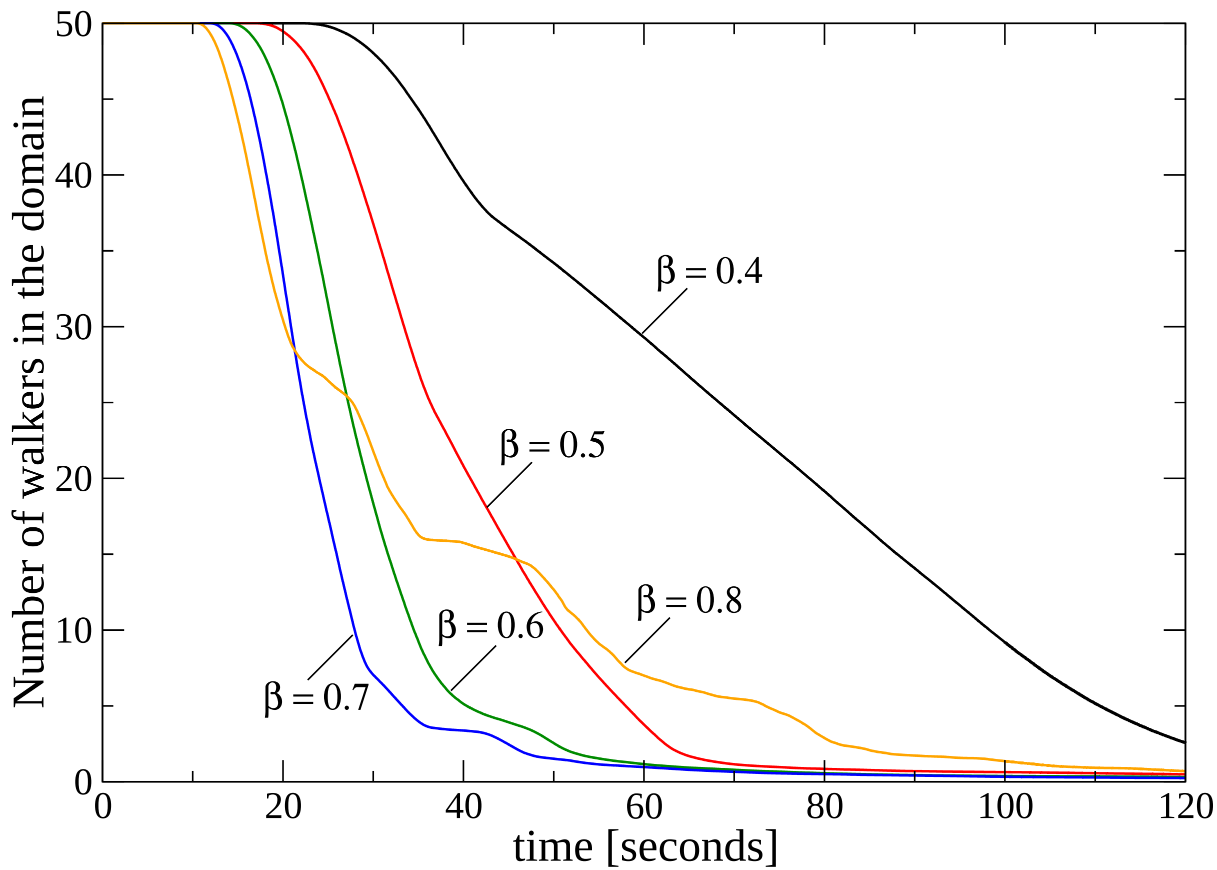


Dynamics through complex venues



3.10. Validation and Analytic Challenges

The role of the "selfishness" parameter Panic: *Breakdown of ordered, cooperative behavior*



3.11. Validation and Analytic Challenges

The invention of a mathematical theory goes through the search for new mathematical structures. Indeed, as observed by Gromov:

> *Mathematics is about "interesting structures". What makes a structure interesting is an abundance of interesting problems; we study a structure by solving these problems. The worlds of science, as well as of mathematics itself, is abundant with gems (germs?) of simple beautiful ideas. When and how many of these ideas direct you toward beautiful mathematics?*

• M. Gromov, In a search for a structure, Part 1: On entropy, http://www.ihes.fr/ gromov/PDF/structre-search-entropy-july5-2012.

This statement indicates how the quest for new methods should end up with the design of new mathematical structures, which might even be richer than what is needed for a specific modeling project. Therefore, mathematicians are motivated to investigate all properties of the new structures.

3.12. Validation and Analytic Challenges

Some challenging problems

- Modeling need a computational approach, Deterministic computational methods are not sufficient, while statistical methods should be developed;
- Modeling at the microscopic scale need to be consistent with the micro-scale dynamics used in the kinetic theory approach;
- Interaction rules evolve in time and propagate in space;
- Understanding the role of learning dynamics;
- The derivation of macroscopic (hydrodynamic) models from the underlying description the kinetic theory approach;

• N. Bellomo and A. Bellouquid, and D. Knopoff, From the micro-scale to collective crowd dynamics, *SIAM Multiscale Model. Simul.*, **11** (2013), 943–963.

• N. Bellomo and A. Bellouquid, On multiscale models of pedestrian crowds - From mesoscopic to macroscopic, *Comm. Math. Sci.*, **13** (2015), 1649–1664.

The End

Thank You!